

New states of quantum matter and where to find them

Quantum matter:

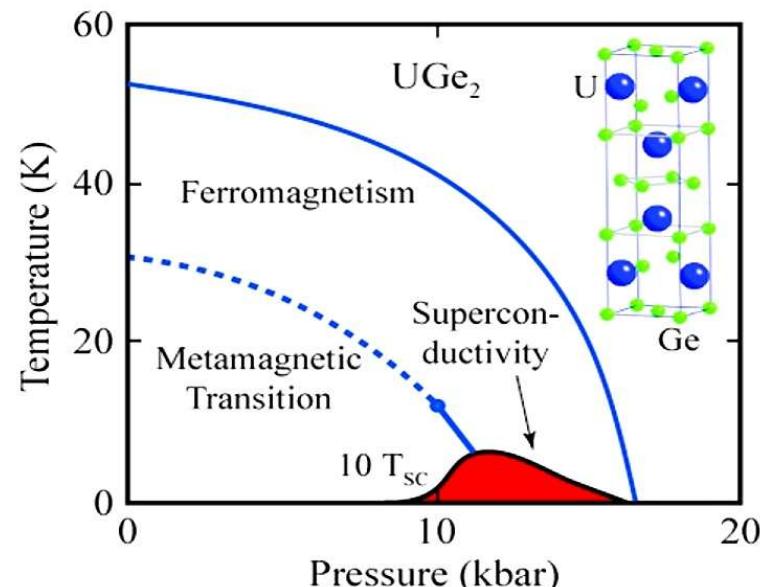
- matter with emerging **macroscopic** properties that are intrinsically **quantum** (superconductors, superfluids, fractional quantum Hall states, spin liquids)



at low temperatures

$$F = E - TS$$

- thermal motion is suppressed
- new types of order can form

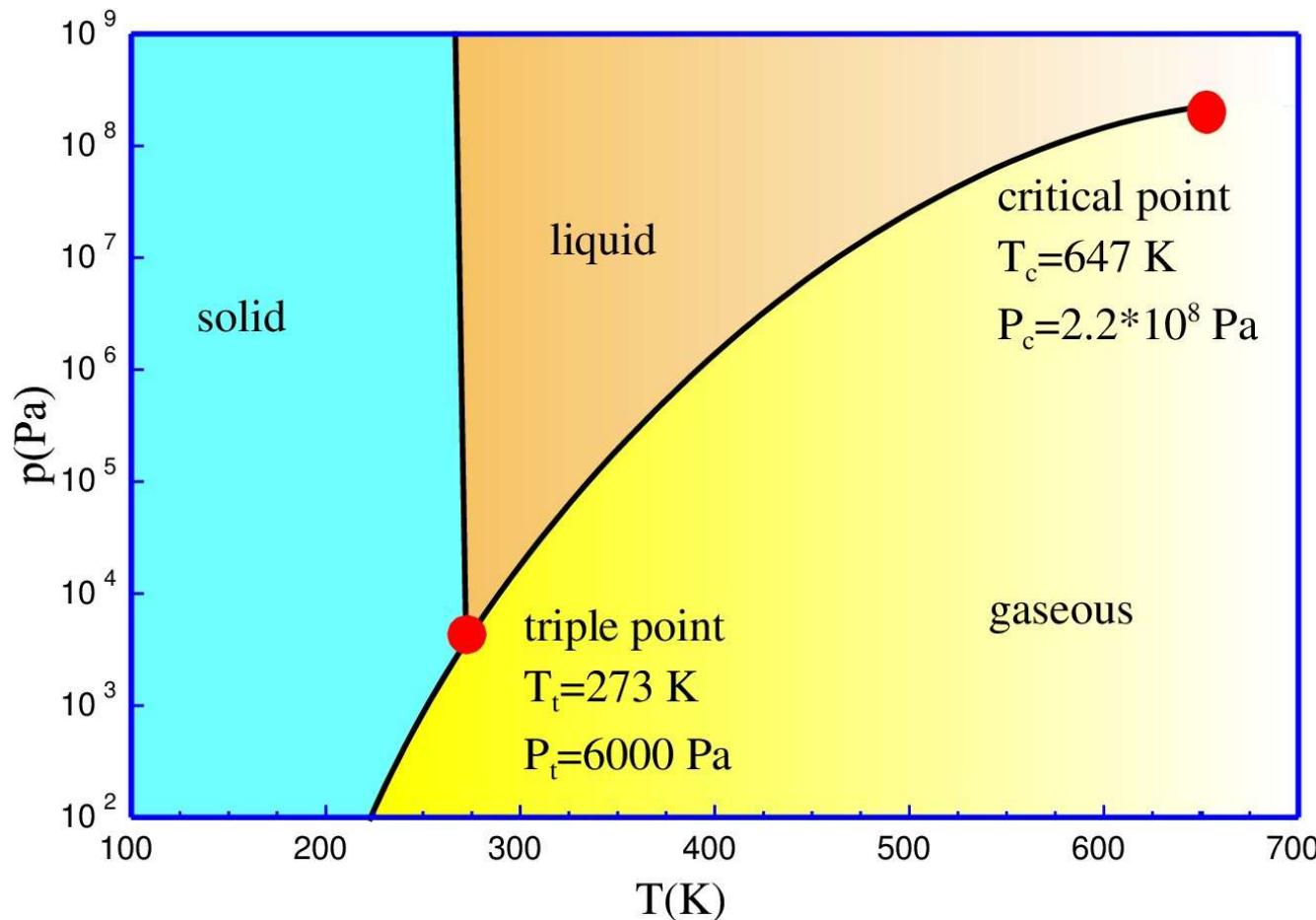


at boundaries of existing phases

- two types of order compete, suppress each other
- novel type of order may appear

- Condensed matter physics: complexity and emerging phenomena
- **Phase transitions and quantum phase transitions**
 - Novel phases close to quantum critical points

Phase diagram of water



Phase transition:

singularity in thermodynamic quantities as functions of external parameters

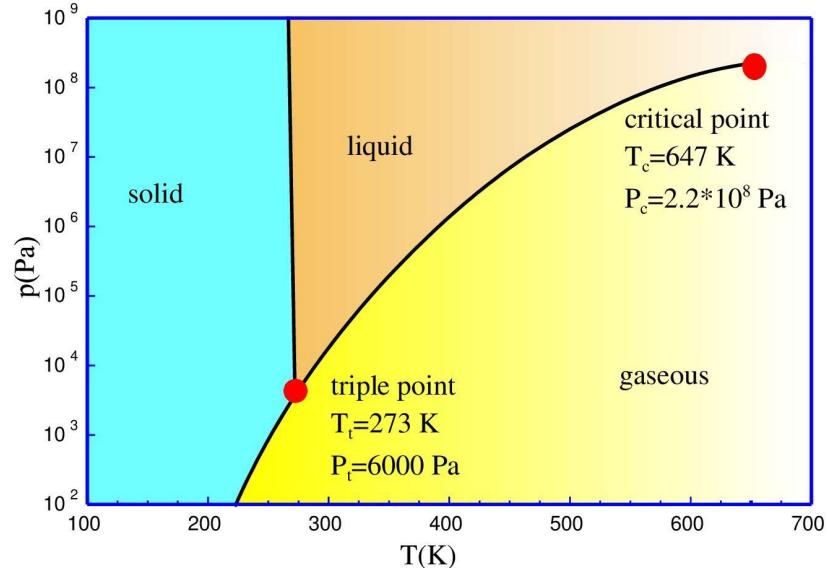
Phase transitions: 1st order vs. continuous

1st order phase transition:

phase coexistence, latent heat,
short range spatial and time correlations

Continuous transition (critical point):

no phase coexistence, no latent heat,
infinite range correlations of fluctuations



Critical behavior at continuous transitions:

diverging correlation length $\xi \sim |T - T_c|^{-\nu}$ and time $\xi_\tau \sim \xi^z \sim |T - T_c|^{-\nu z}$

- Manifestation: critical opalescence (Andrews 1869)

Universality: critical exponents are independent of microscopic details

Critical opalescence

Binary liquid system:

e.g. hexane and methanol

$T > T_c \approx 36^\circ\text{C}$: fluids are miscible

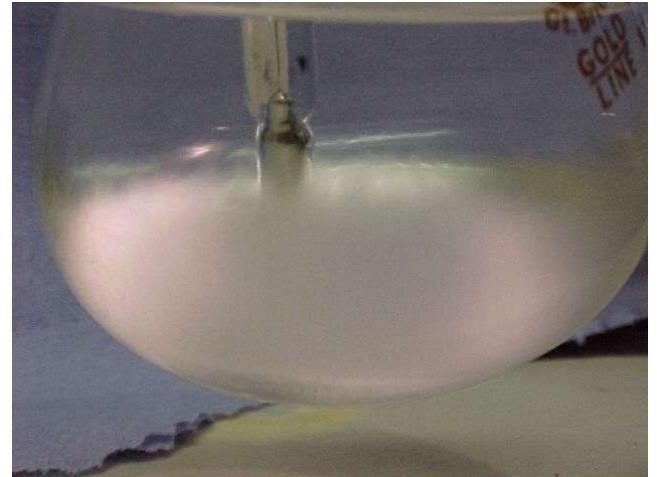
$T < T_c$: fluids separate into two phases

$T \rightarrow T_c$: length scale ξ of fluctuations grows

When ξ reaches the scale of a fraction of a micron (wavelength of light):

strong light scattering
fluid appears milky

46°C



39°C



18°C



How important is quantum mechanics close to a critical point?

Two types of fluctuations:

thermal fluctuations (**thermal motion**), energy scale $k_B T$

quantum fluctuations (**quantum zero-point motion**), energy scale $\hbar\omega_c$

Quantum effects **unimportant** if $\hbar\omega_c \ll k_B T$.

Critical slowing down:

$$\omega_c \sim 1/\xi_\tau \sim |T - T_c|^{\nu z} \rightarrow 0 \text{ at the critical point}$$

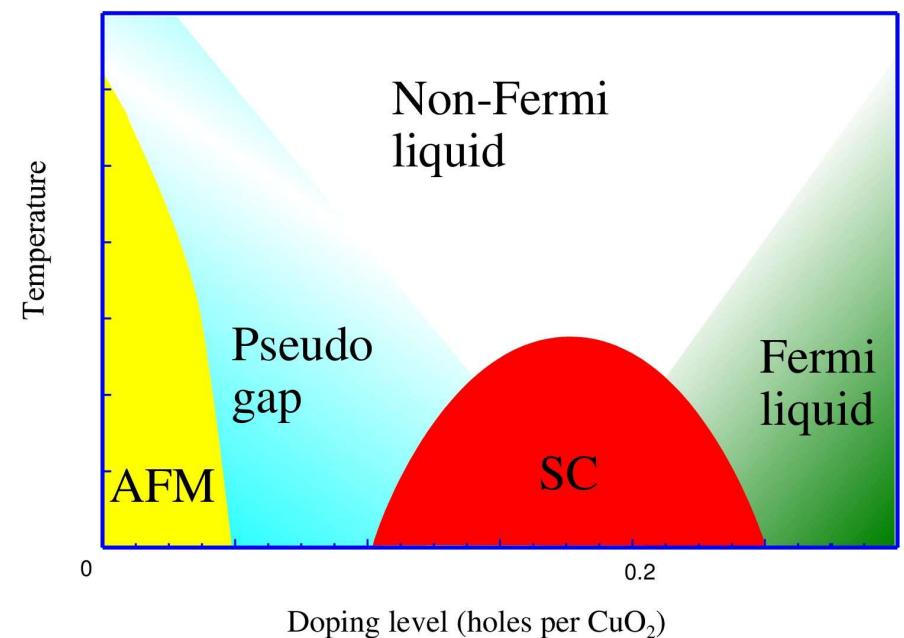
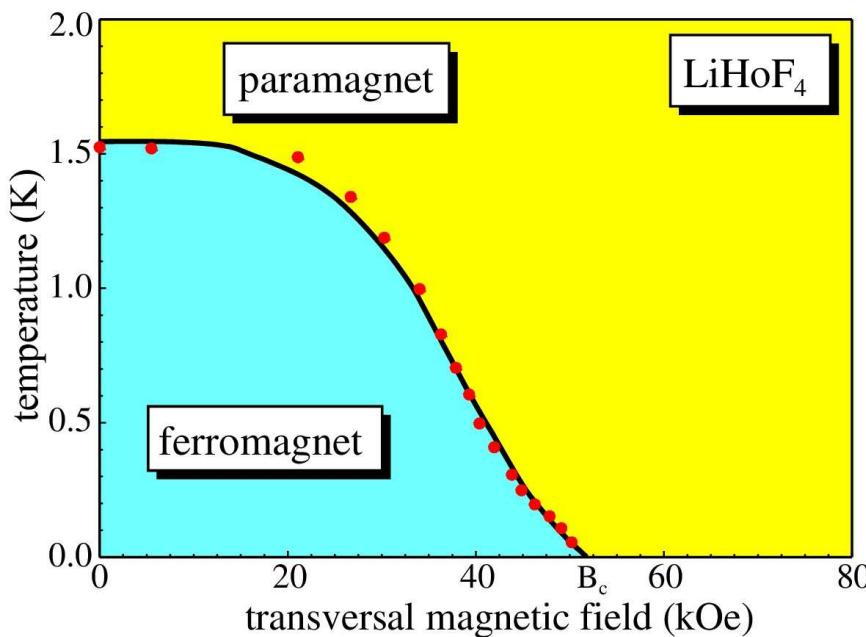
- ⇒ For any **nonzero** temperature, quantum fluctuations do **not** play a role close to the critical point
- ⇒ Quantum fluctuations **do** play a role at **zero** temperature

Zero-temperature continuous phase transitions constitute a special class of phase transitions, they are intrinsically quantum in nature

Quantum phase transitions

occur at **zero temperature** as function of pressure, magnetic field, chemical composition, ...

driven by **quantum zero-point motion** rather than thermal fluctuations

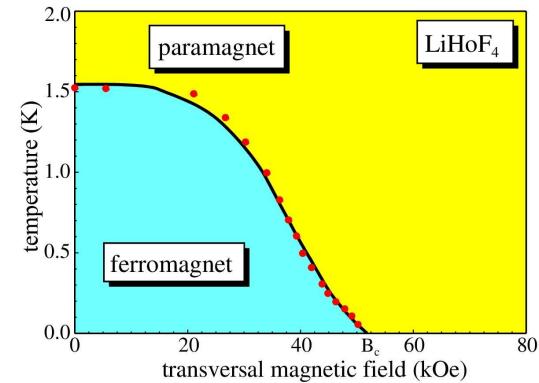


Phase diagrams of LiHoF_4 and a typical high- T_c superconductor such as $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$

Toy model: transverse field Ising model

Quantum spins S_i on a lattice: (c.f. LiHoF_4)

$$\begin{aligned} H &= -J \sum_i \mathbf{S}_i^z \mathbf{S}_{i+1}^z - h \sum_i \mathbf{S}_i^x \\ &= -J \sum_i \mathbf{S}_i^z \mathbf{S}_{i+1}^z - \frac{h}{2} \sum_i (\mathbf{S}_i^+ + \mathbf{S}_i^-) \end{aligned}$$



J : exchange energy, favors parallel spins, i.e., ferromagnetic state

h : transverse magnetic field, induces quantum fluctuations between up and down states, favors paramagnetic state

Limiting cases:

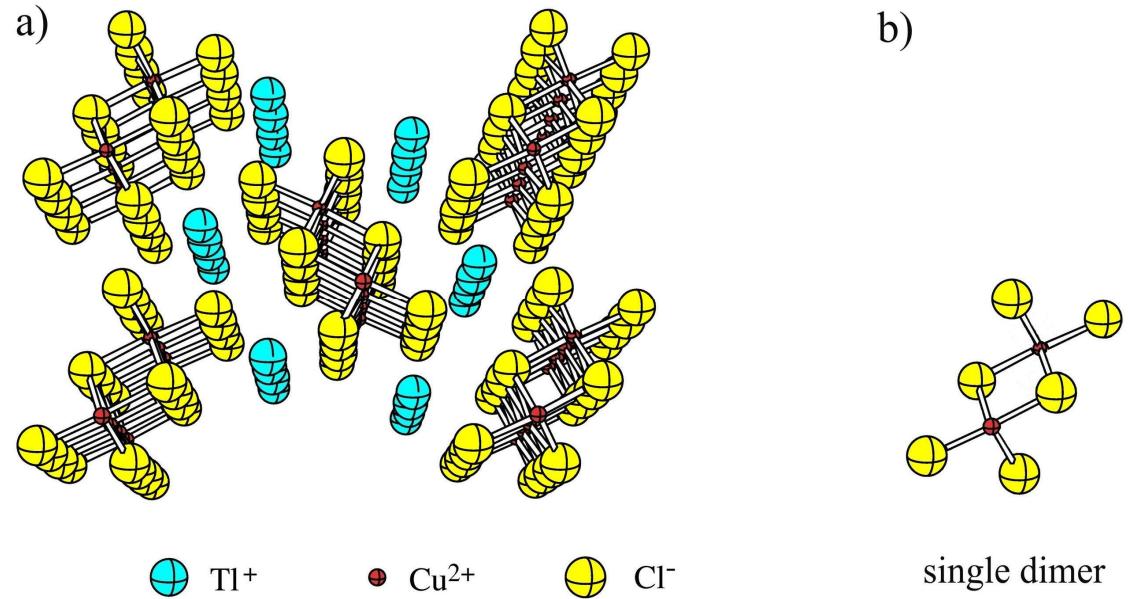
$|J| \gg |h|$ ferromagnetic ground state as in classical Ising magnet

$|J| \ll |h|$ paramagnetic ground state as for independent spins in a field

⇒ Quantum phase transition at $|J| \sim |h|$ (in 1D, transition is at $|J| = |h|$)

Magnetic quantum critical points of TiCuCl_3

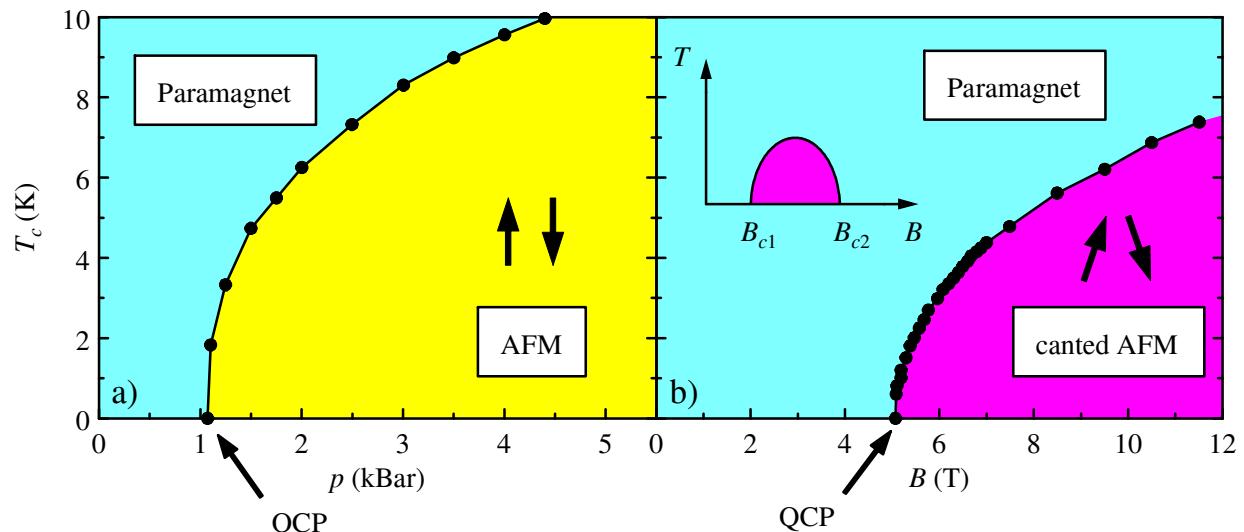
- TiCuCl_3 is magnetic insulator
- planar Cu_2Cl_6 dimers form infinite double chains
- Cu^{2+} ions carry spin-1/2 moment



antiferromagnetic order

can be induced by

- applying pressure
- applying a magnetic field



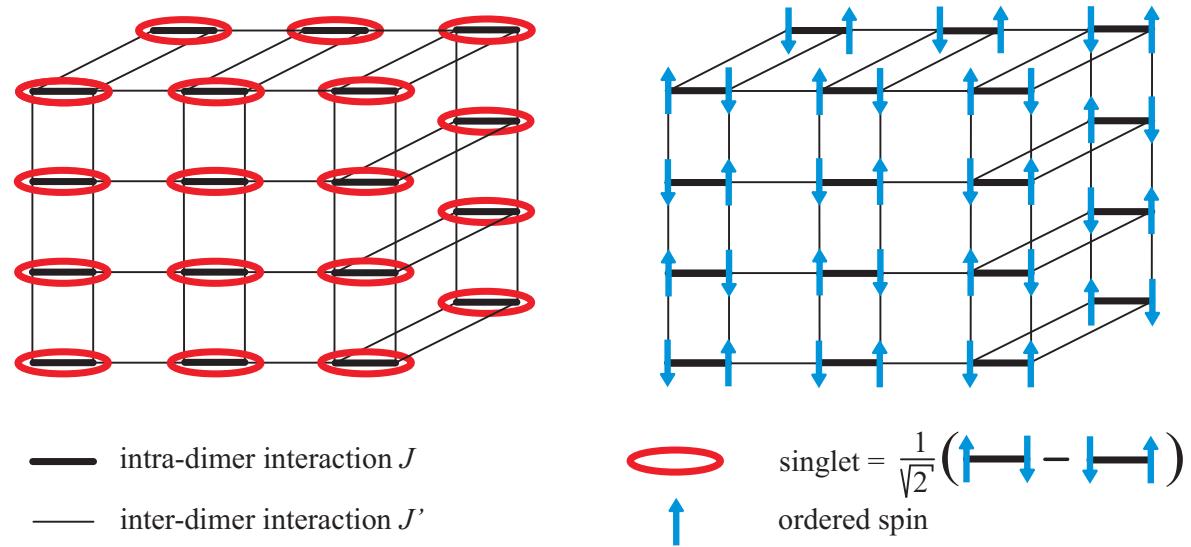
Pressure-driven quantum phase transition in TlCuCl_3

quantum Heisenberg model

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j - \vec{h} \cdot \sum_i \vec{S}_i$$

$$J_{ij} = \begin{cases} J & \text{intra-dimer} \\ J' & \text{between dimers} \end{cases}$$

pressure changes ratio J/J'



Limiting cases:

$|J| \gg |J'|$ spins on each dimer form singlet \Rightarrow no magnetic order

low-energy excitations are “triplons” (single dimers in the triplet state)

$|J| \approx |J'|$ long-range antiferromagnetic order (Néel order)

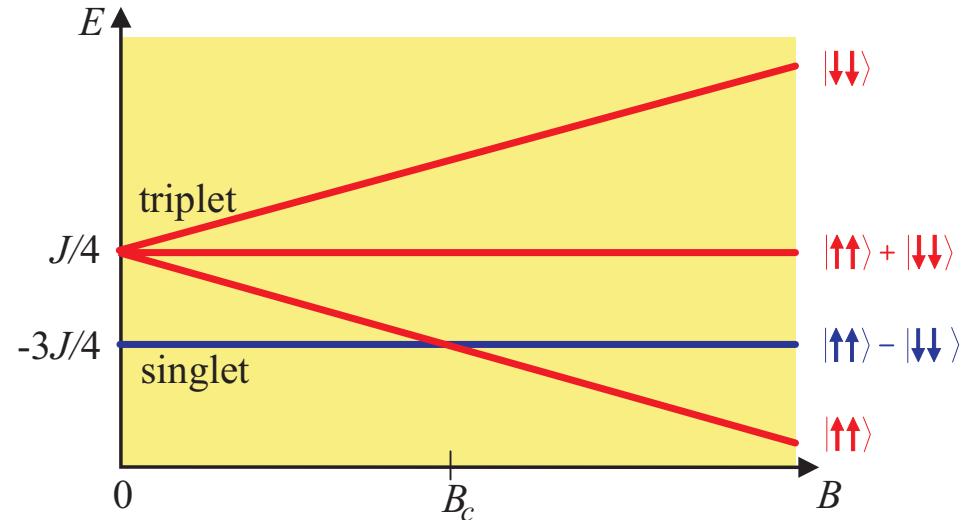
low-energy excitations are long-wavelength spin waves

\Rightarrow quantum phase transition at some critical value of the ratio J/J'

Field-driven quantum phase transition in TiCuCl_3

Single dimer in field:

- field does not affect singlet ground state but splits the triplet states
- ground state: singlet for $B < B_c$ and (fully polarized) triplet for $B > B_c$

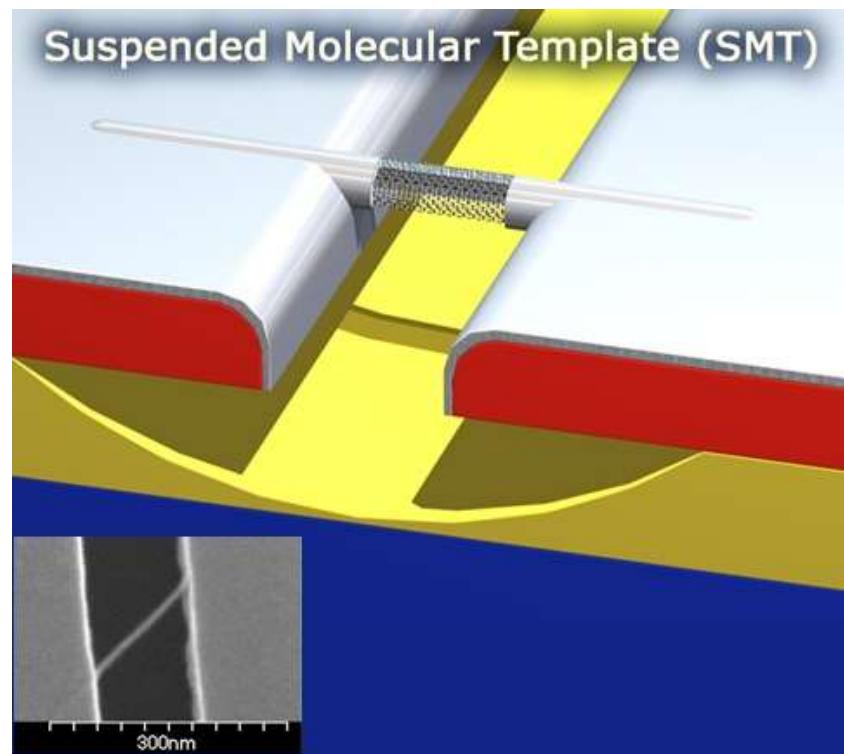
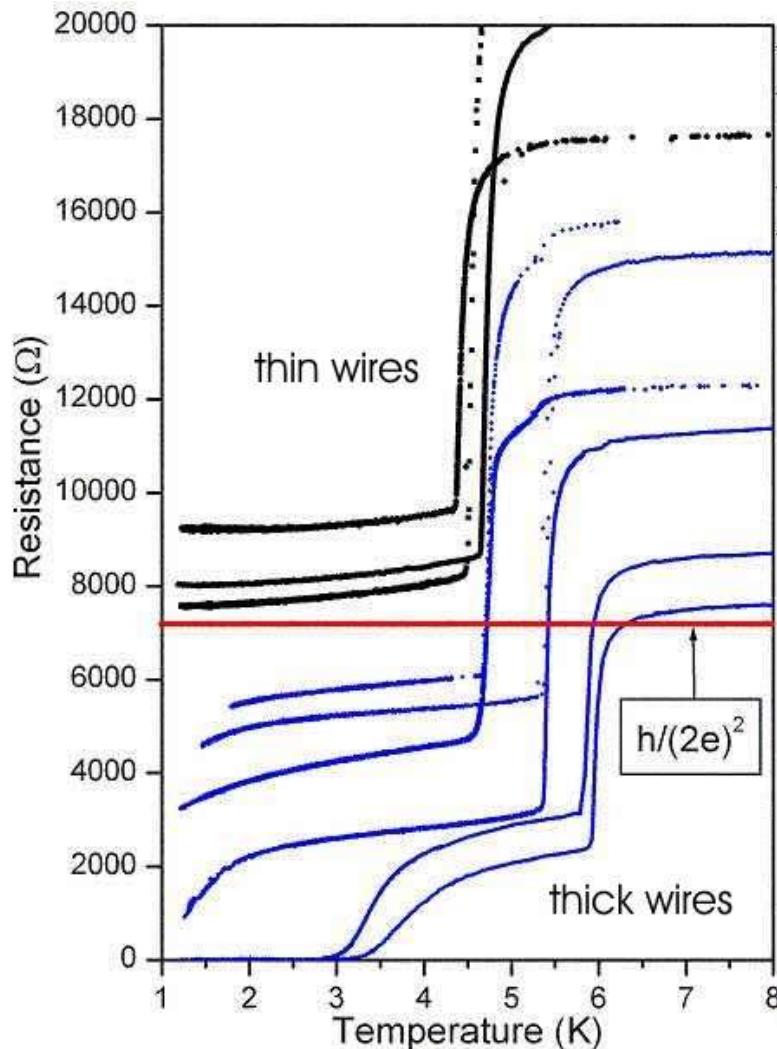


Full Hamiltonian:

- singlet-triplet transition of isolated dimer splits into two transitions
- at B_{c1} , triplon gap closes, system is driven into ordered state (uniform magnetization \parallel to field and antiferromagnetic order \perp to field)
- “canted” antiferromagnet is Bose-Einstein condensate of triplons
- at B_{c2} system enters fully polarized state

Superconductor-metal QPT in ultrathin nanowires

- ultrathin MoGe wires (width ~ 10 nm)
- produced by molecular templating using a single carbon nanotube

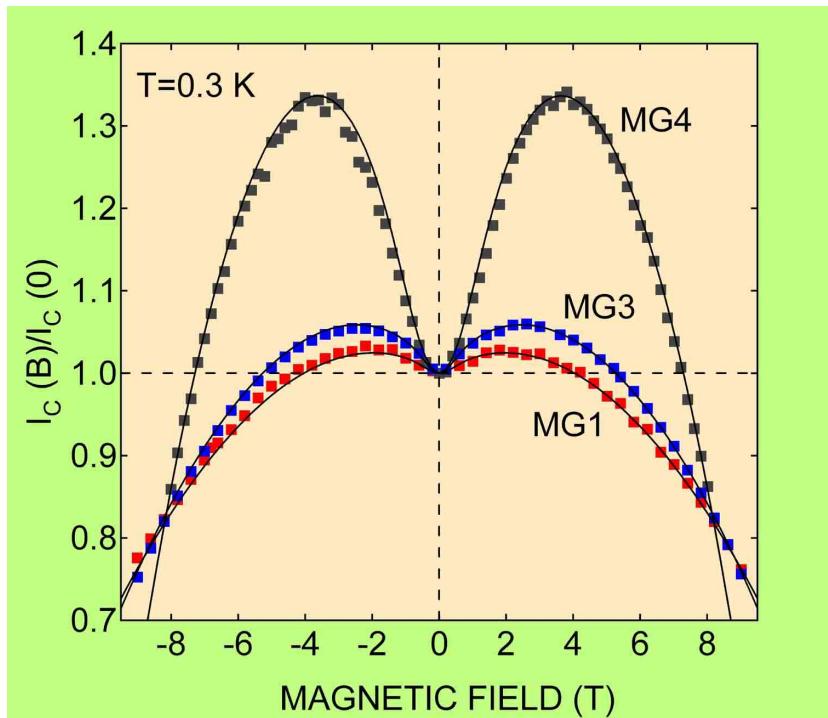


- thicker wires are superconducting at low temperatures
- thinner wires remain metallic

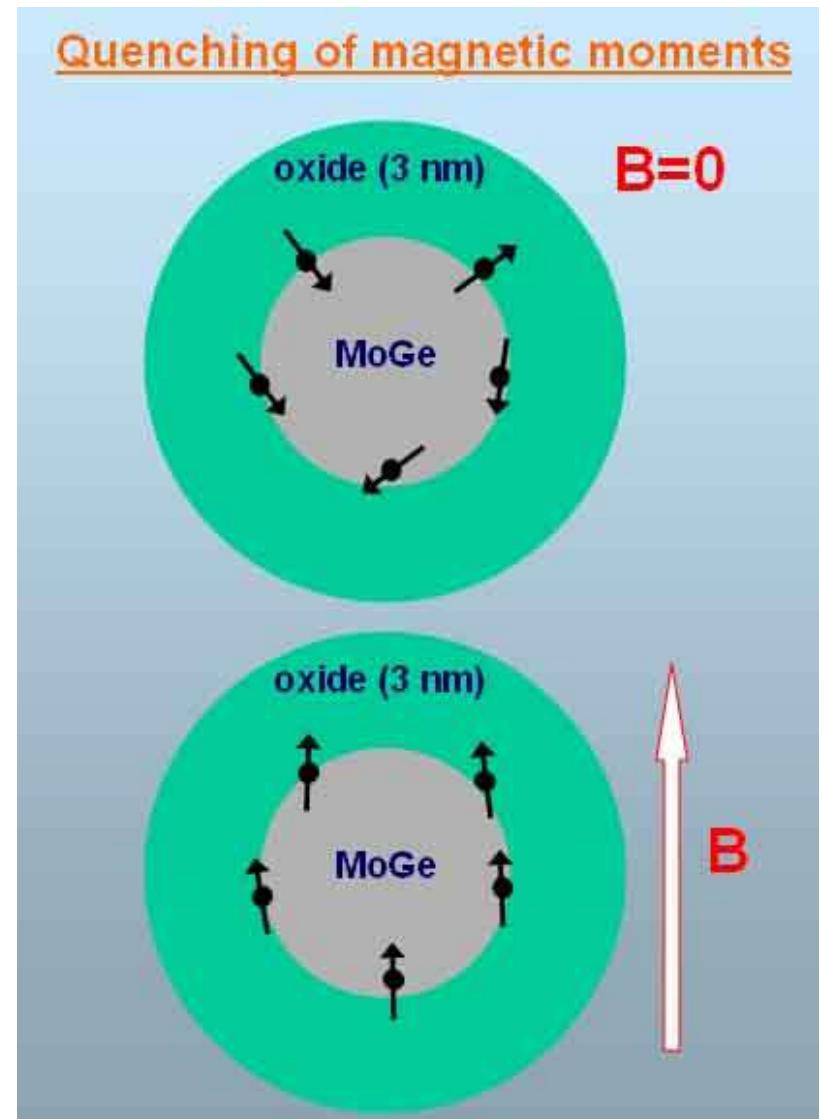
superconductor-metal QPT as function of wire thickness

Pairbreaking mechanism

- pair breaking by surface magnetic impurities
- random impurity positions
⇒ quenched **disorder**
- gapless excitations in metal phase
⇒ Ohmic **dissipation**

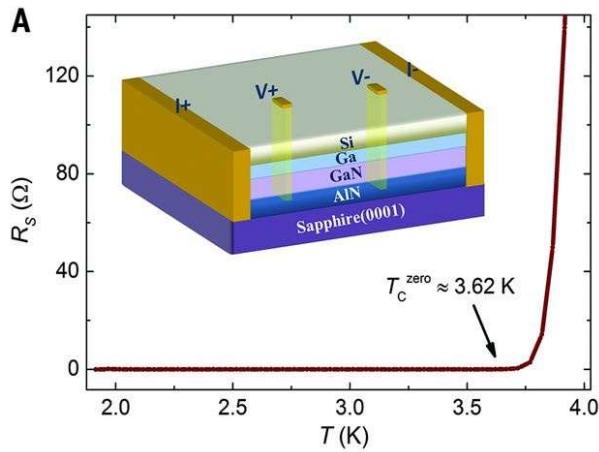


weak field enhances superconductivity



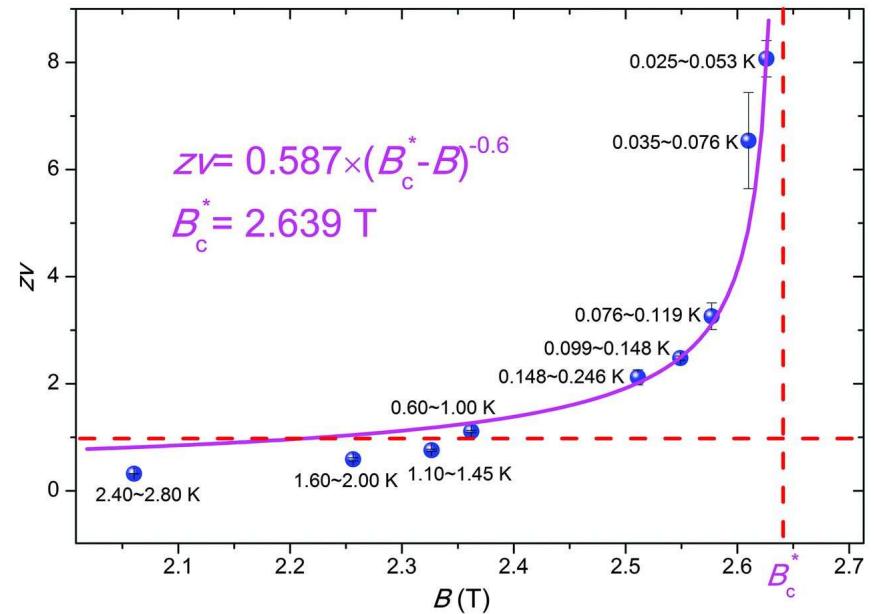
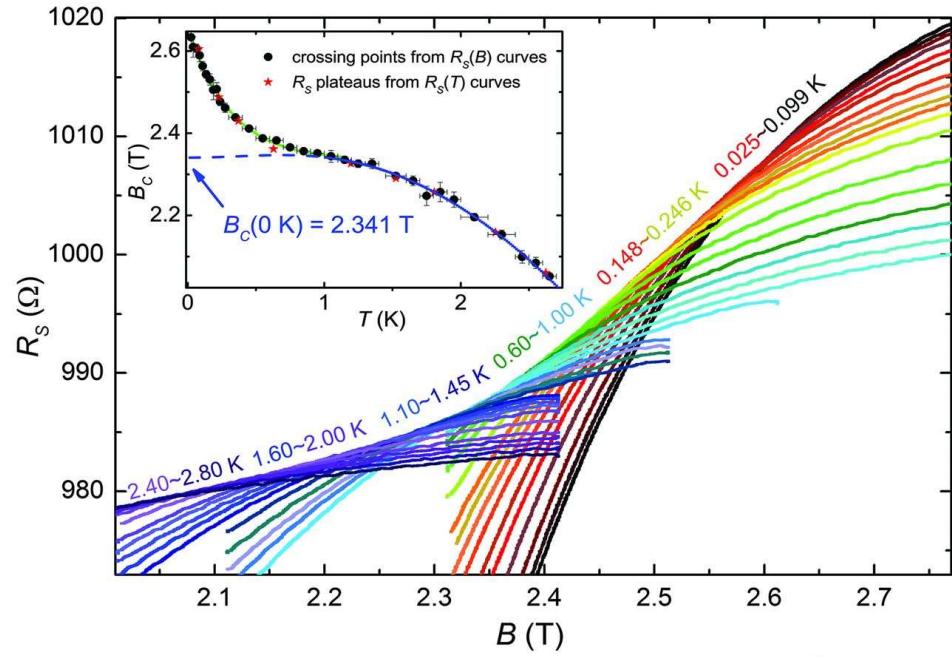
magnetic field aligns the impurities and reduces magnetic scattering

Experiment: Ga thin films

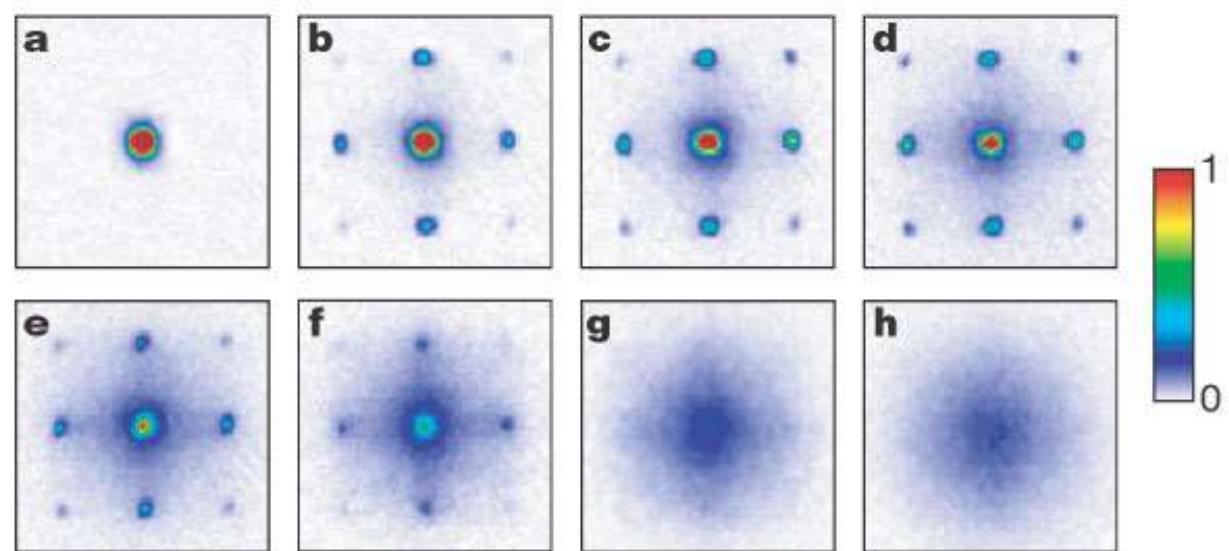
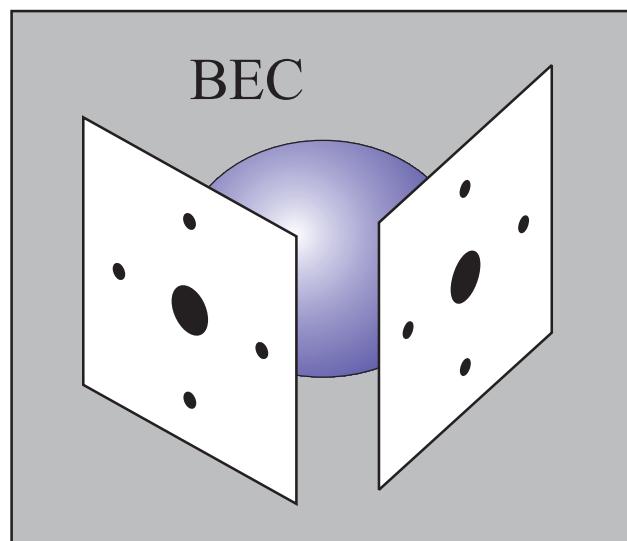
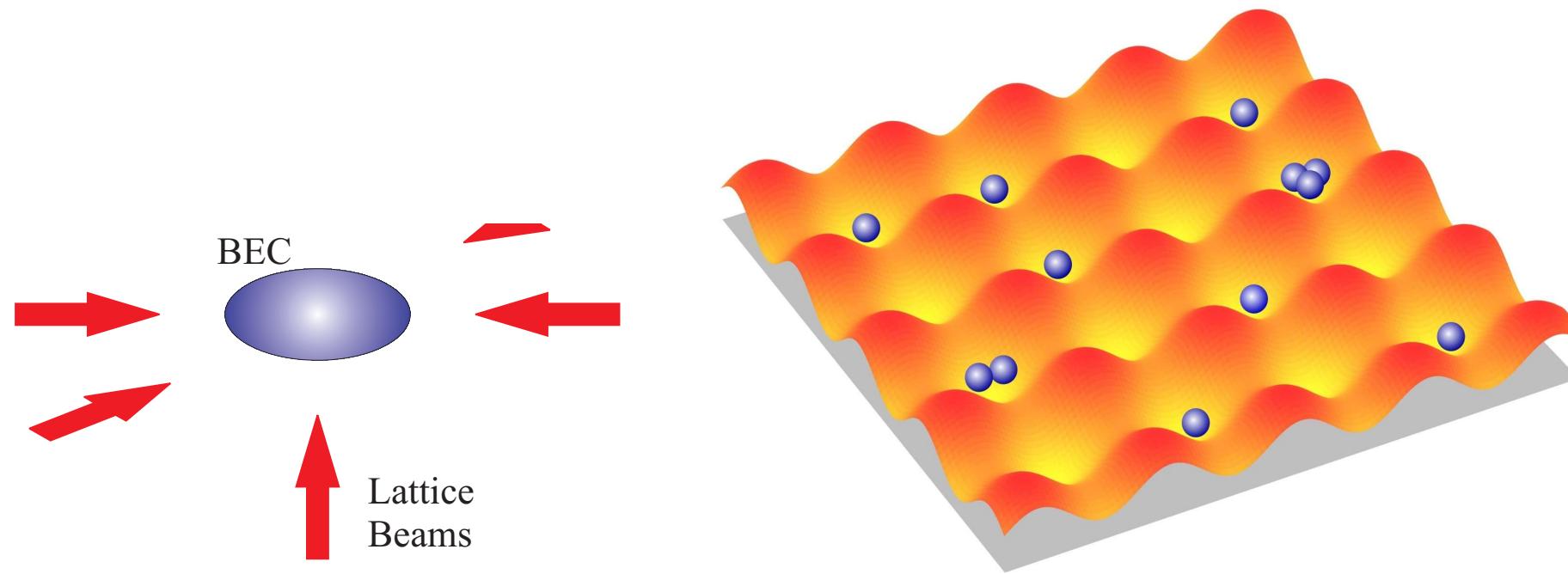


Xing et al., Science 350, 542 (2015)

- three-monolayer Ga films
- superconductivity below $T_c \approx 3.62 \text{ K}$, suppressed by magnetic field
- field-driven QPT well described by **2D infinite-randomness critical point**
- dynamical exponent **diverges** as $z \sim |B - B_c|^{-\nu\psi}$ with $\nu \approx 1.2, \psi \approx 0.5$



Mott transition in a Bose-Einstein condensate

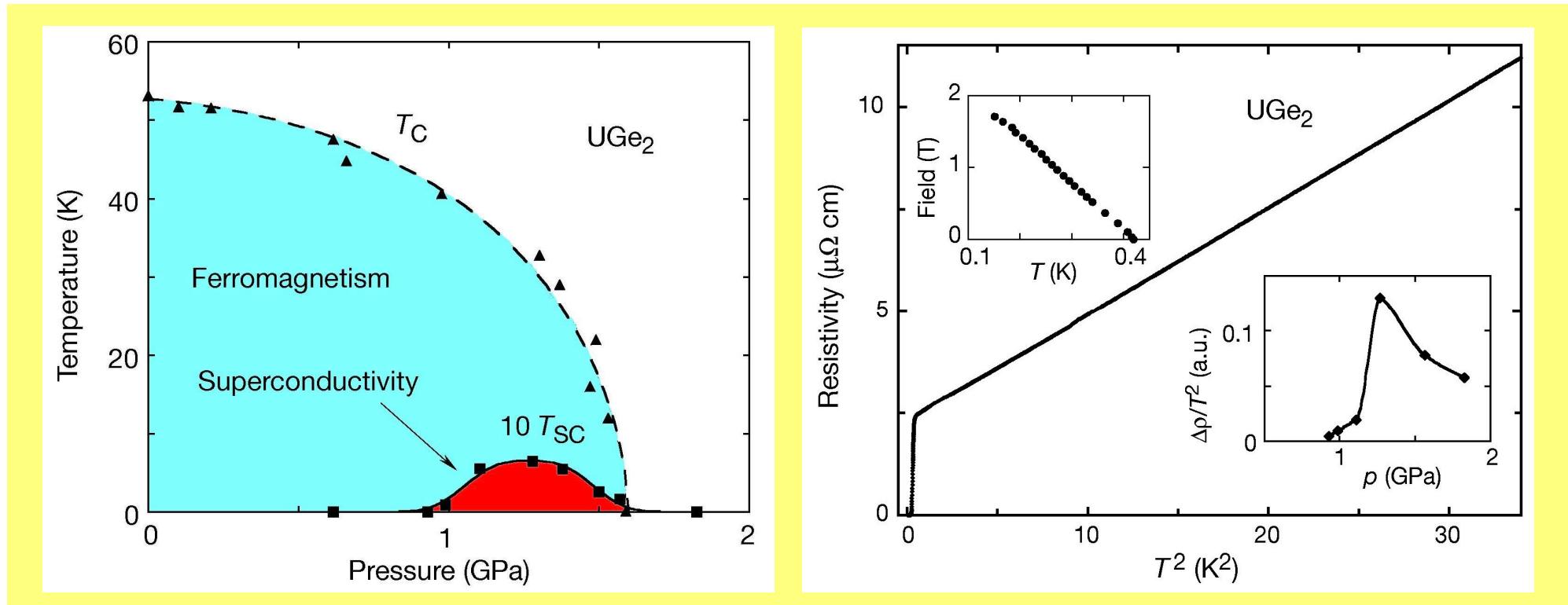


- Condensed matter physics: complexity and emerging phenomena
 - Phase transitions and quantum phase transitions
 - **Novel phases close to quantum critical points**

Exotic superconductivity in UGe_2

Phase diagram:

- phase diagram of UGe_2 has pocket of **superconductivity** close to ferromagnetic quantum phase transition (electrical resistivity **vanishes** below about 1K)
- in this pocket, UGe_2 is **ferromagnetic and superconducting** at the same time
- superconductivity appears only in superclean samples



Character of superconductivity in UGe_2

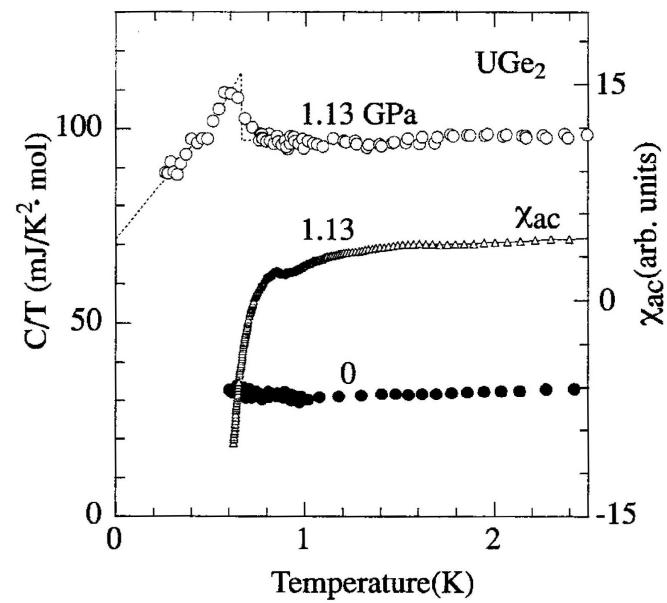
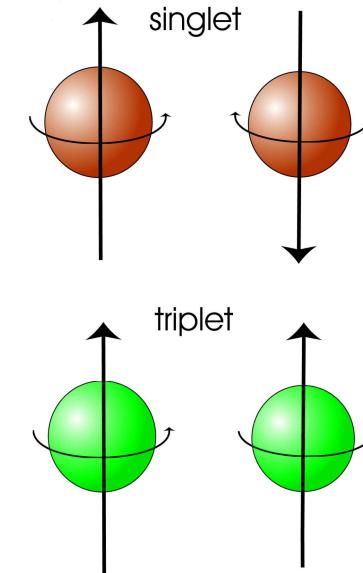
not compatible with conventional (BCS) superconductivity:

- in superconductor, electrons form (Cooper) pairs of spin-up and spin-down electrons
- ferromagnetism requires majority of spins to be in one direction

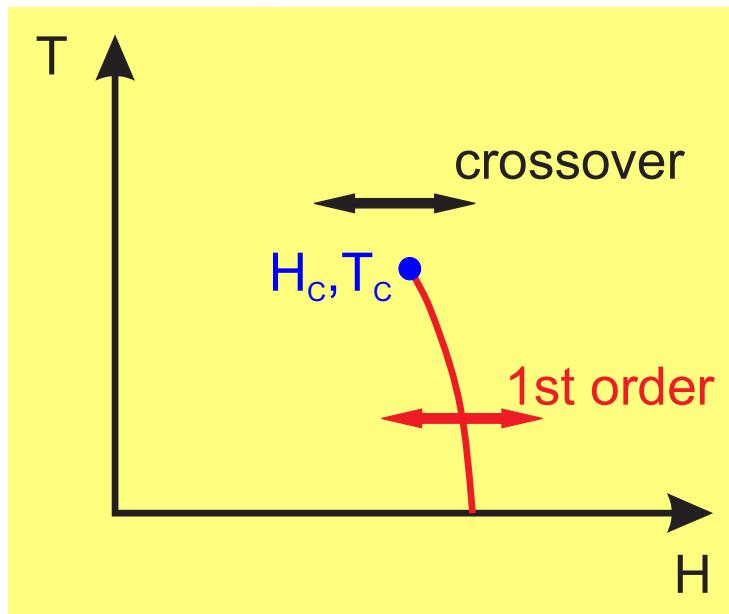
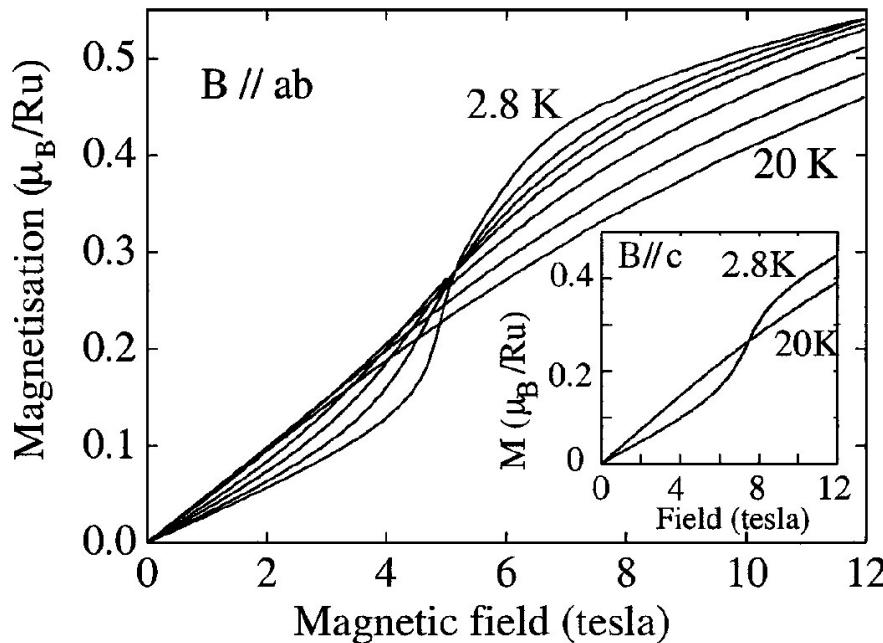
theoretical ideas:

- phase separation (layering or disorder): **NO!**
- partially paired FFLO state: **NO!**
- spin triplet pairs with odd spatial symmetry, magnetic fluctuations promote this type of pairing

Magnetic quantum phase transition induces spin-triplet superconductivity



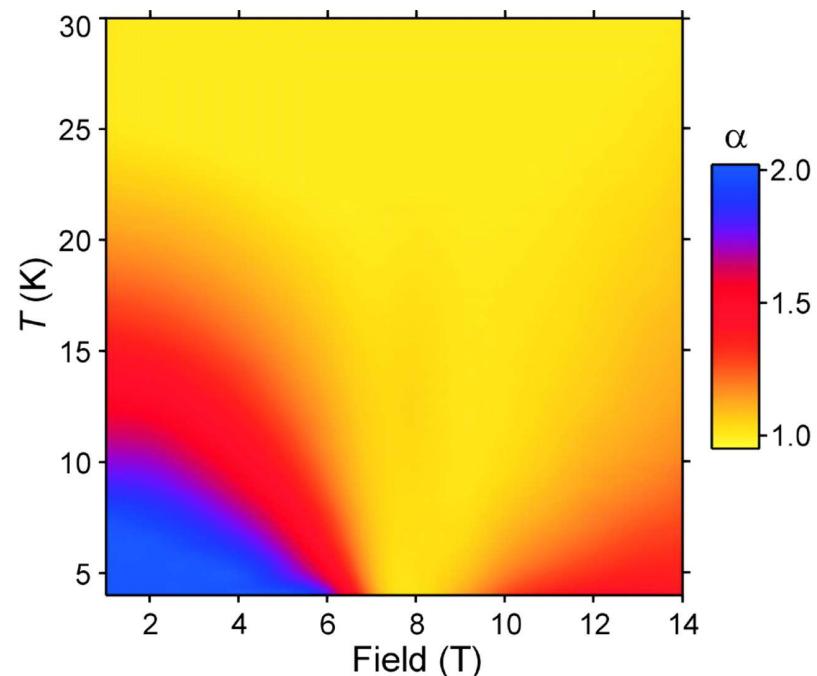
Metamagnetic transition in $\text{Sr}_3\text{Ru}_2\text{O}_7$



- $\text{Sr}_3\text{Ru}_2\text{O}_7$ undergoes metamagnetic transition as function of field
- critical endpoint can be tuned to $T = 0$ by tilting the field

⇒ **metamagnetic quantum critical point**

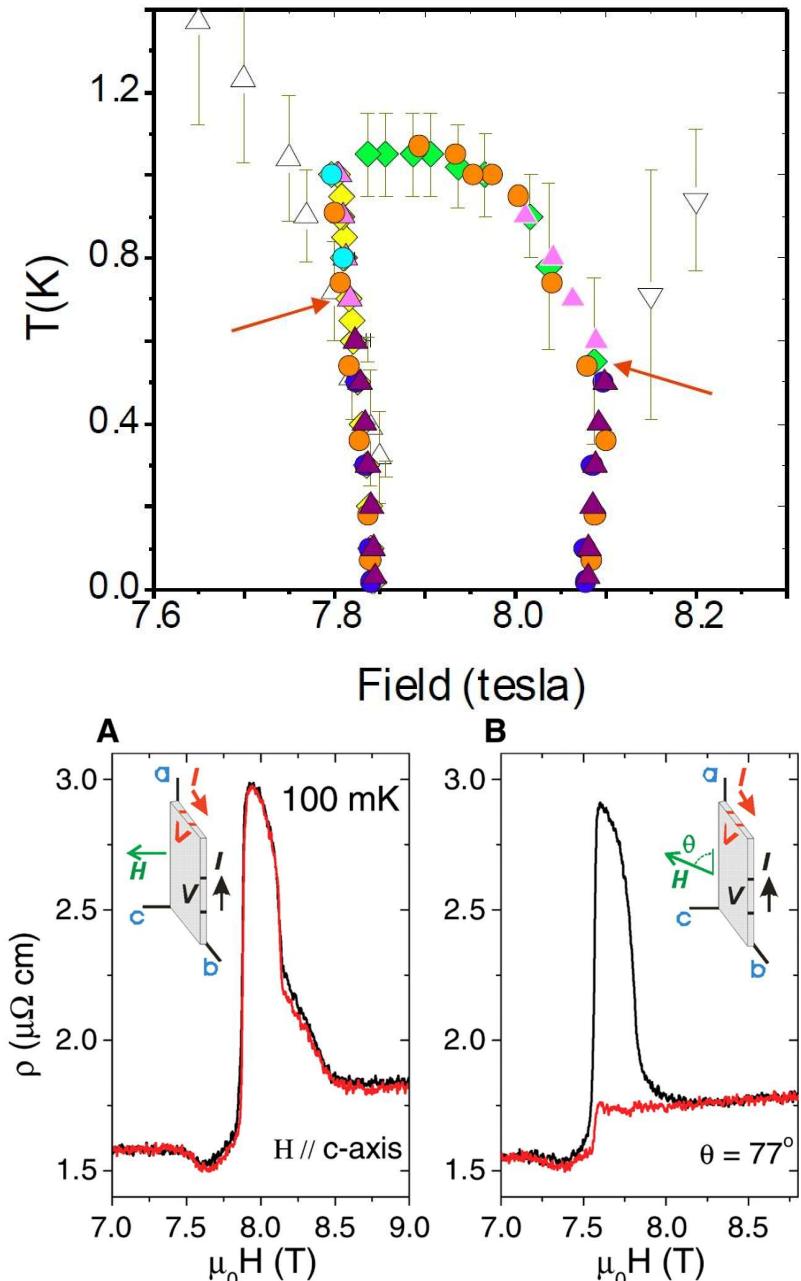
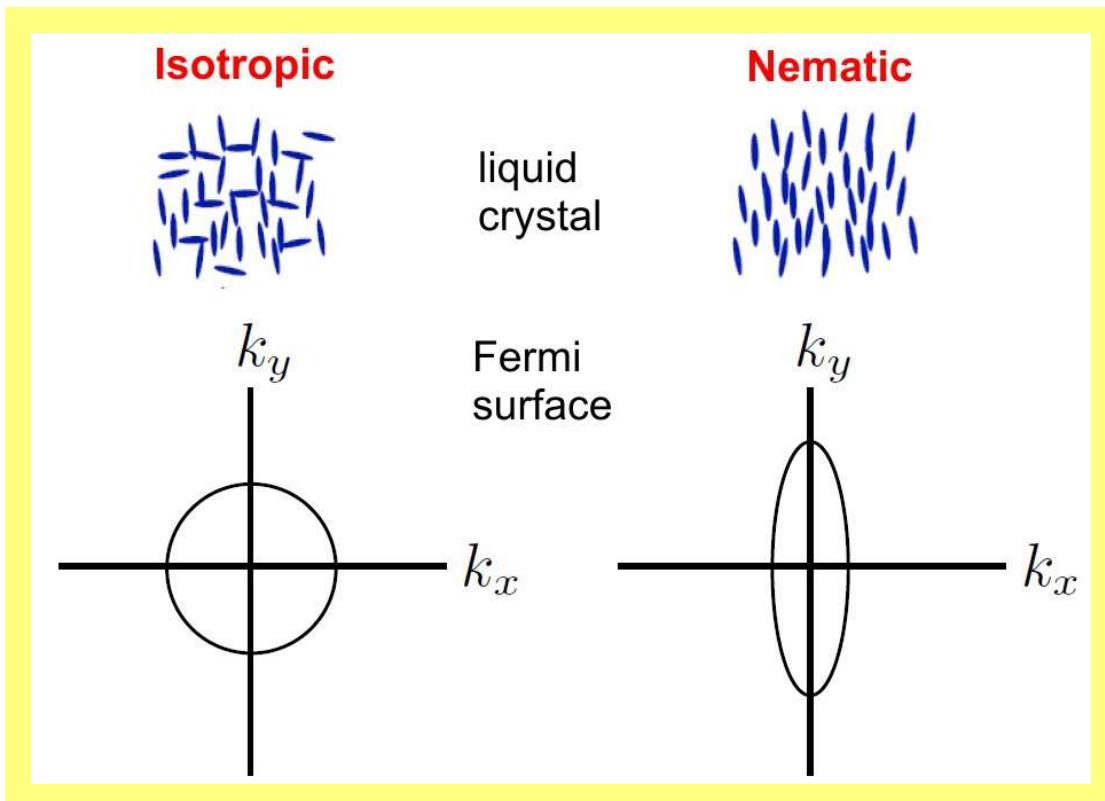
- seen in temperature dependence of resistivity $\rho = \rho_0 + AT^\alpha$



Electronic nematic phase in $\text{Sr}_3\text{Ru}_2\text{O}_7$

- in pure samples at low temperatures,
QCP **preempted by novel phase**
- resistivity highly anisotropic ($\text{C}4 \rightarrow \text{C}2$)

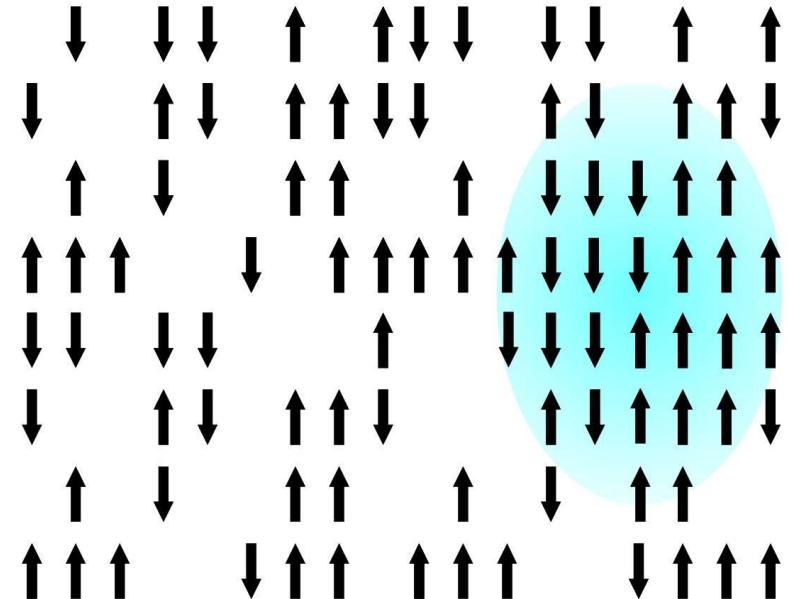
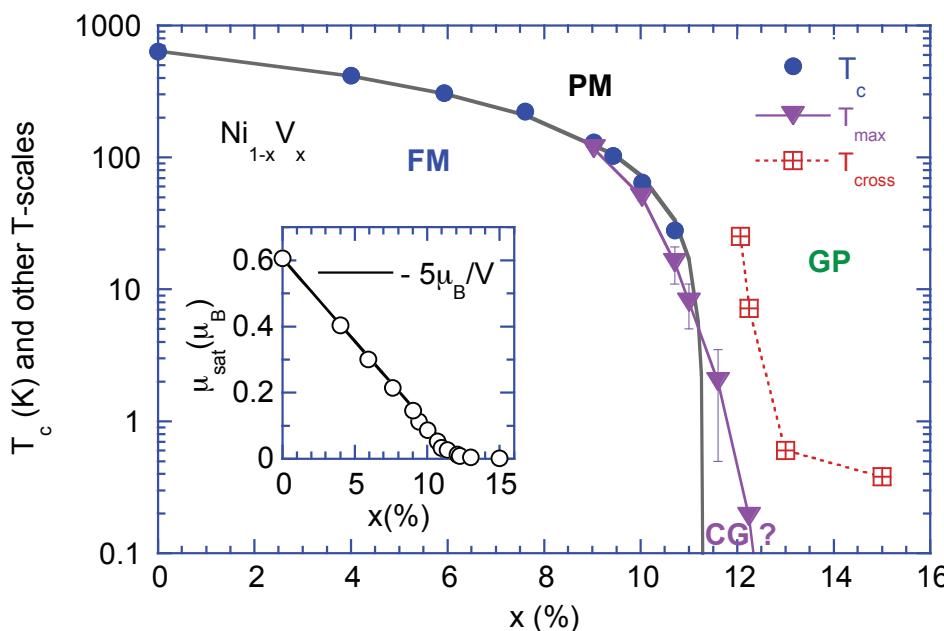
⇒ new phase is **electronic nematic**
(translational invariant but rotational
symmetry spontaneously broken)



Disorder and Griffiths phases

QPT in a disordered system:

- **rare region** can be locally in ordered phase even if bulk system is in disordered phase
- probability of rare region **exponentially small** $p(L) \sim \exp(-cL^d)$:
- rare regions act as large **superspins**
⇒ slow dynamics, large contribution to TD

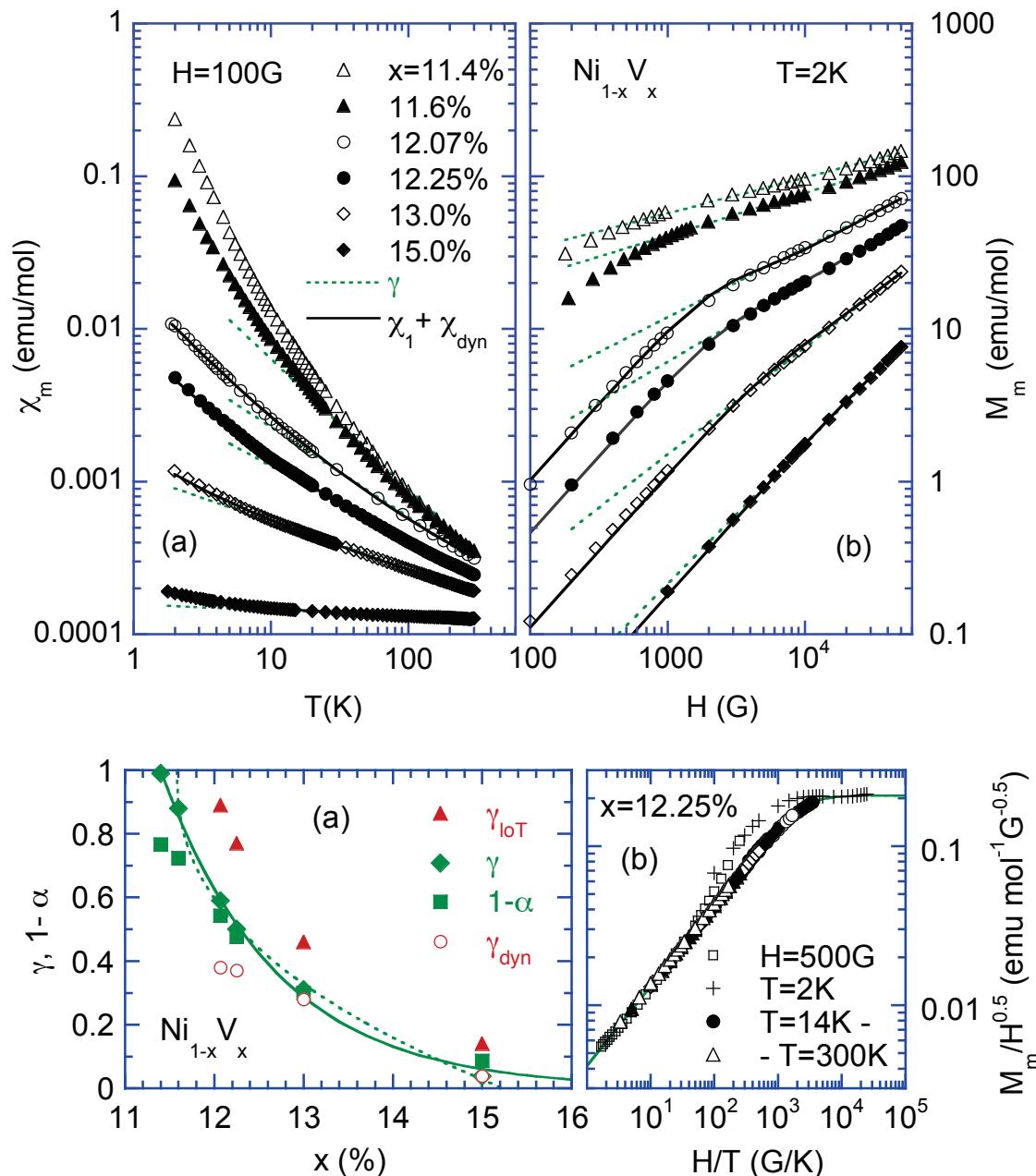


Can rare regions dominate the thermodynamic response?

⇒ **quantum Griffiths phase**

- example:
diluted ferromagnet $\text{Ni}_{1-x}\text{V}_x$

Quantum Griffiths phase in $\text{Ni}_{1-x}\text{V}_x$

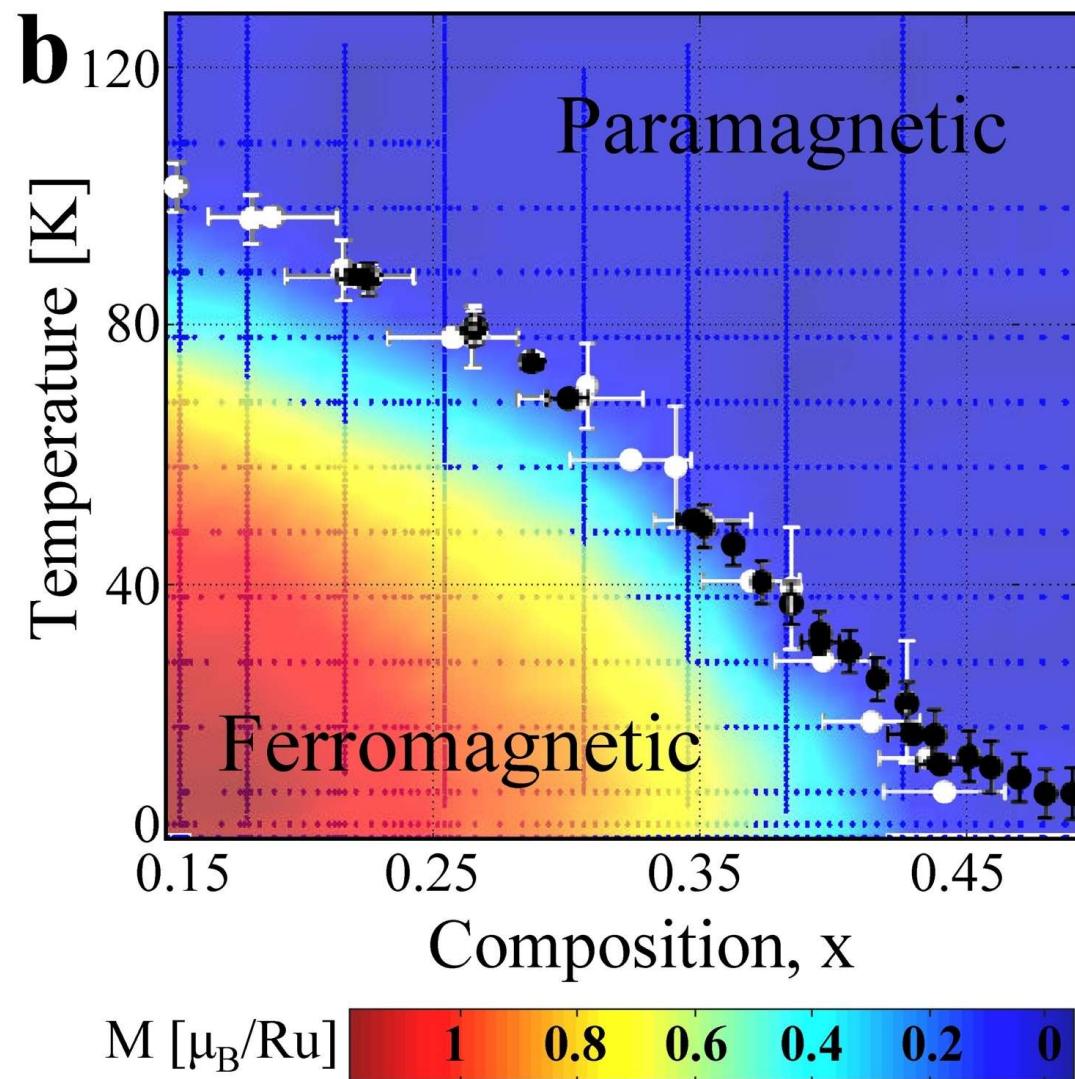
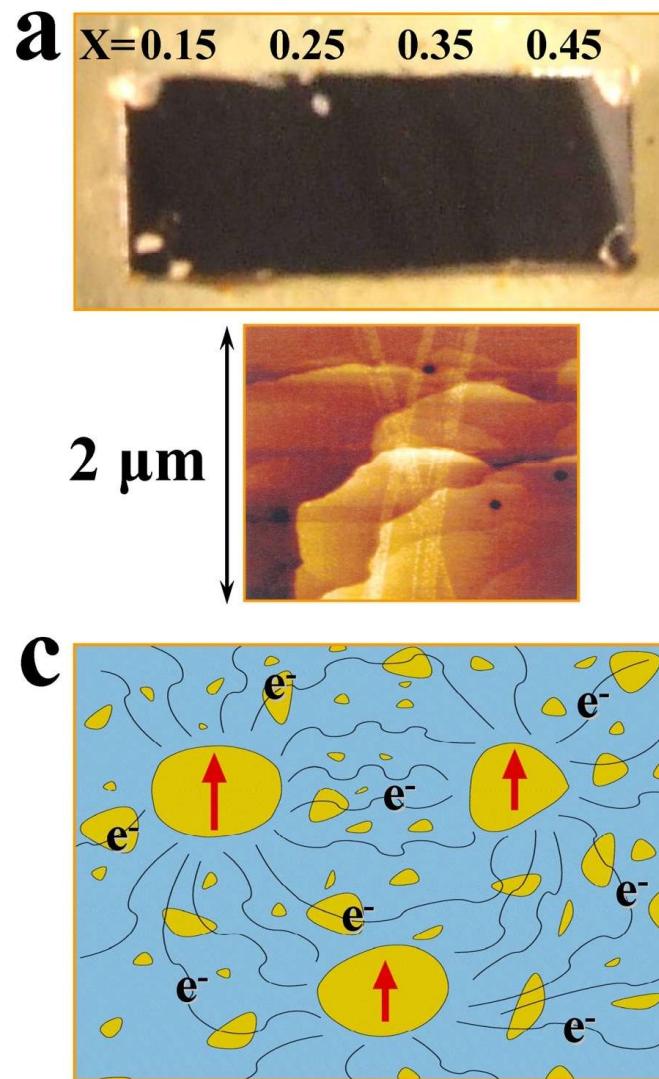


at concentrations above x_c :

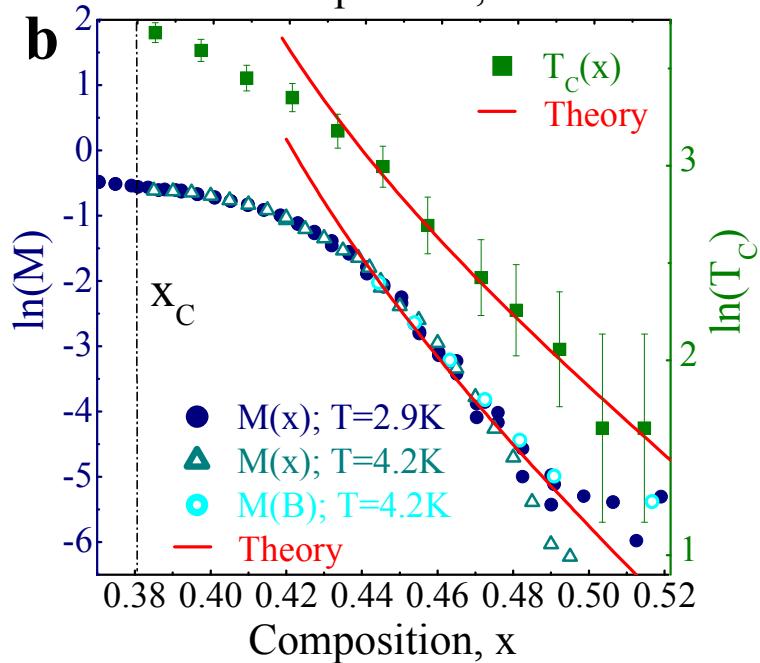
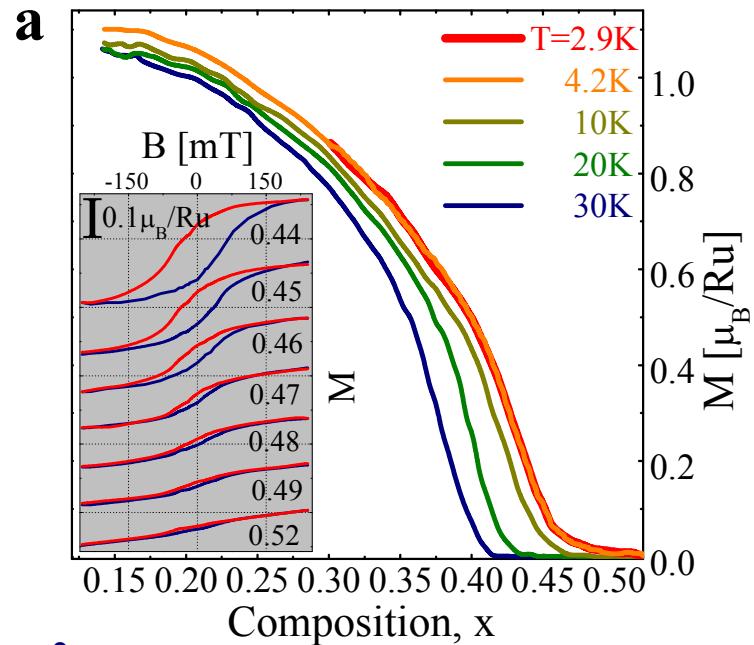
- strongly enhanced magnetic response
- susceptibility $\chi(T)$ diverges as $T \rightarrow 0$
- $\chi(T)$ and $m(H)$ follow nonuniversal power laws $\chi \sim T^{\lambda-1}$, $m \sim H^\lambda$ **(Griffiths singularities)**
- Griffiths exponent $\lambda = 1 - \gamma$ varies systematically with x
- experiments agree with **infinite-randomness** critical point scenario

quantum Griffiths phase for $x \approx 11.5$ to 15%.

Rare regions and smeared phase transition in $\text{Sr}_{1-x}\text{Ca}_x\text{RuO}_3$



Composition-tuned smeared phase transitions

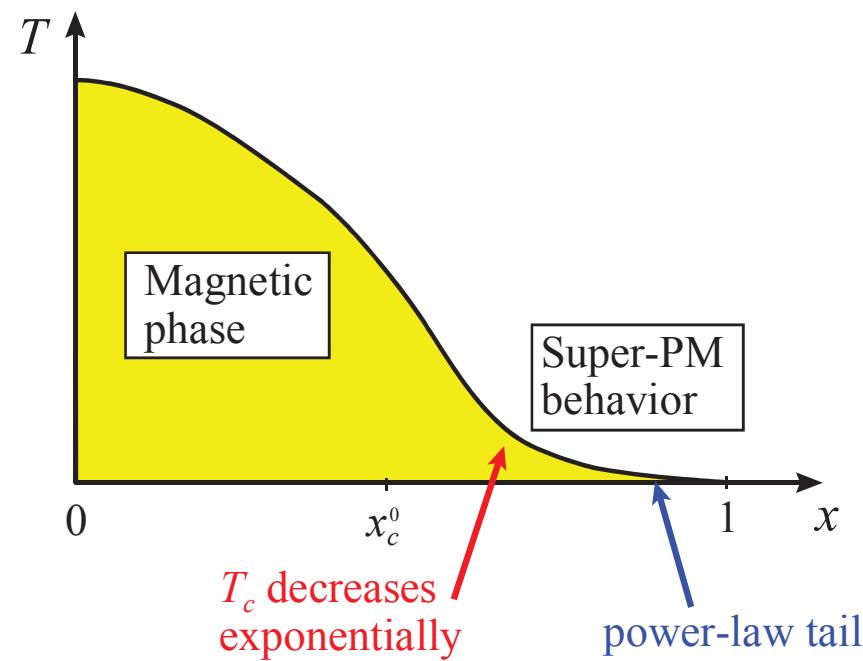


Magnetization and T_c in tail:

$$M, T_c \sim \exp \left[-C \frac{(x - x_c^0)^{2-d/\phi}}{x(1-x)} \right]$$

for $x \rightarrow 1$:

$$M, T_c \sim (1-x)^{L_{\min}^d}$$

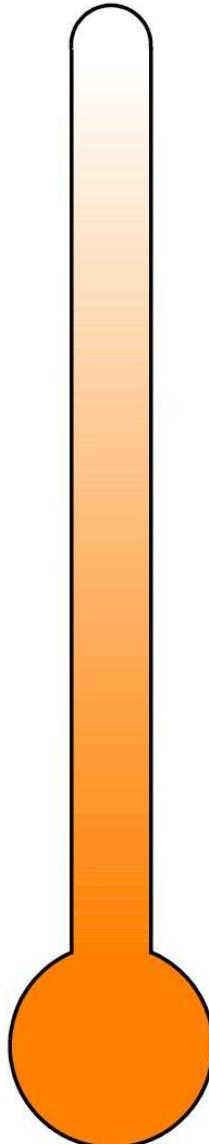


Conclusions

- “**More is different:**” condensed matter physics explores **emerging phenomena** caused by the interplay of many constituent particles
- new states of quantum matter can be found at **low temperatures** and at **boundaries** between existing phases
- quantum phase transitions occur at **zero temperature** as a function of a parameter like pressure, chemical composition, disorder, magnetic field
- quantum phase transitions are caused by **quantum fluctuations** (i.e, Heisenberg's uncertainty principle) rather than thermal fluctuations
- quantum phase transitions can have fascinating consequences including the genesis of **new phases of matter**

Quantum phase transitions provide a novel ordering principle in condensed matter physics

Wonderland at low temperatures



273K (0C)	water freezes
195K (-78C)	carbon dioxide sublimates (dry ice)
133K (-140C)	superconductivity in cuprate perovskites
77K (-196C)	nitrogen (air) liquefies
66K (-207C)	nitrogen (air) freezes
4.2K (-268.9C)	helium liquefies
2.2K (-270.9C)	helium becomes superfluid
170 nK	Bose-Einstein condensation of rubidium
0K (-273.1C)	absolute zero of temperature

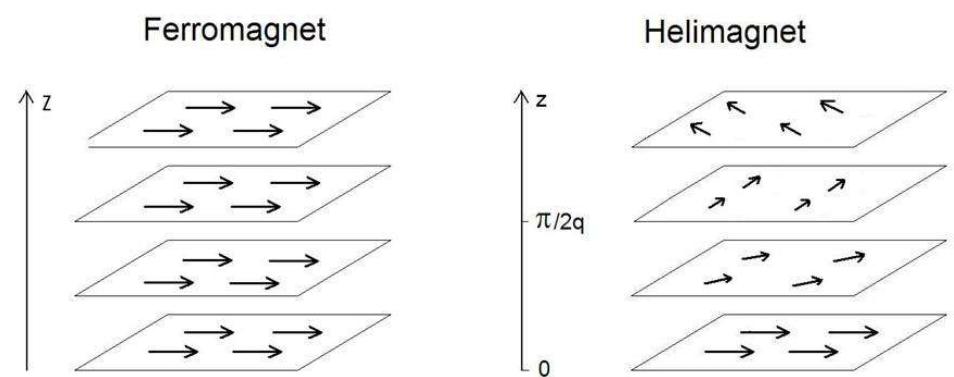
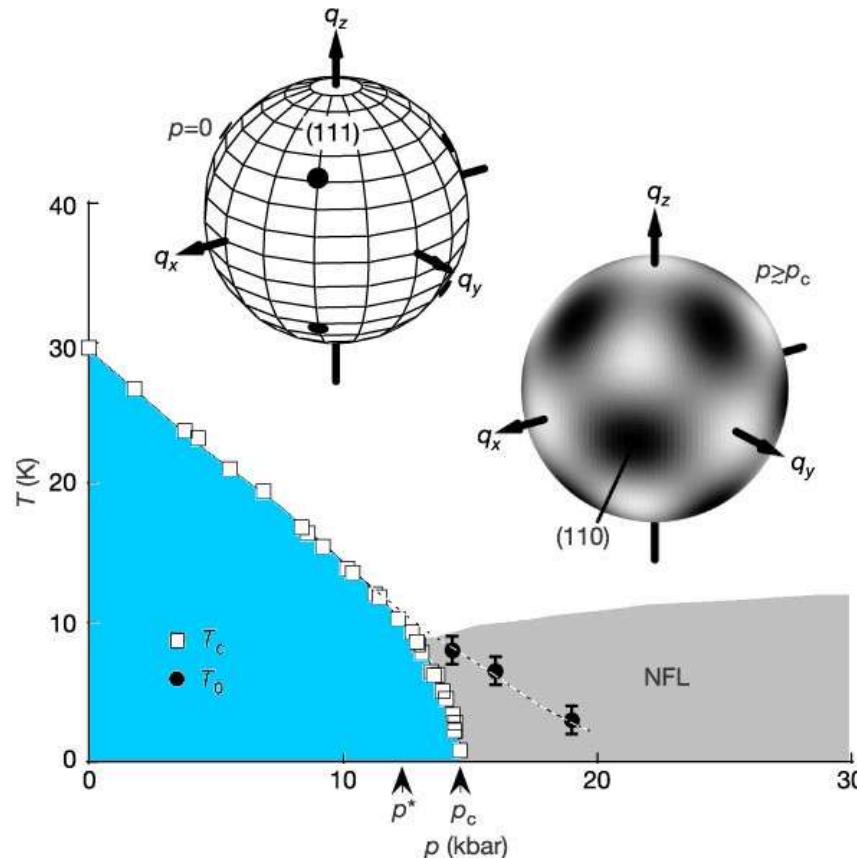
Magnetic phases in MnSi

Phase diagram: (Pfleiderer et al, 2004)

- magnetic transition at 30 K at ambient pressure
- transition tunable by hydrostatic pressure
- quantum phase transition at $p_c = 14$ kbar

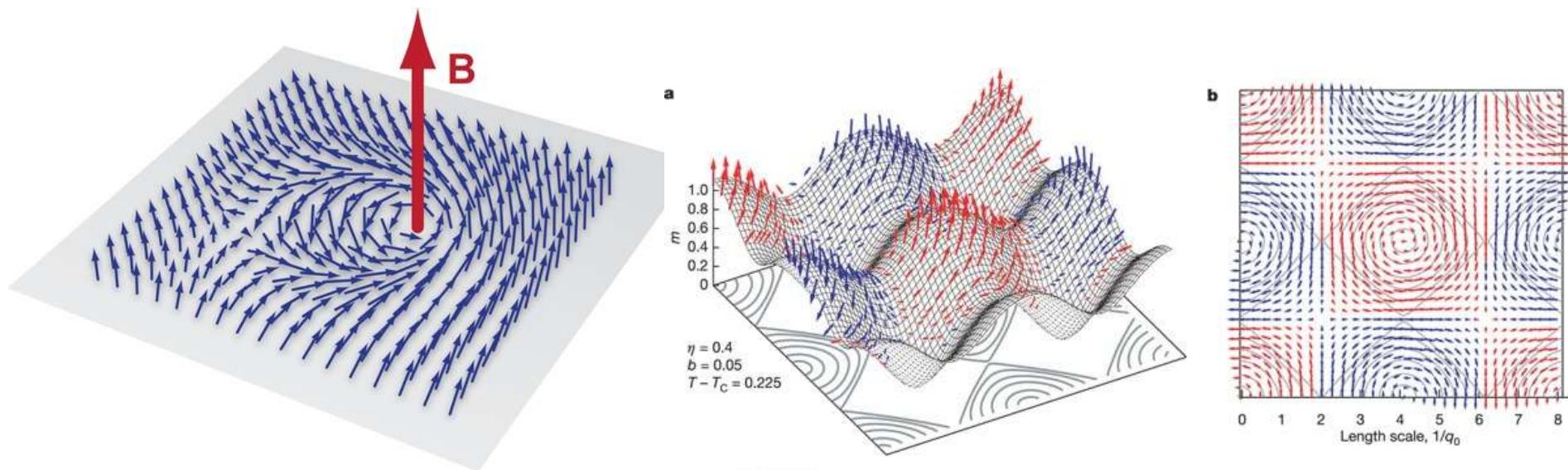
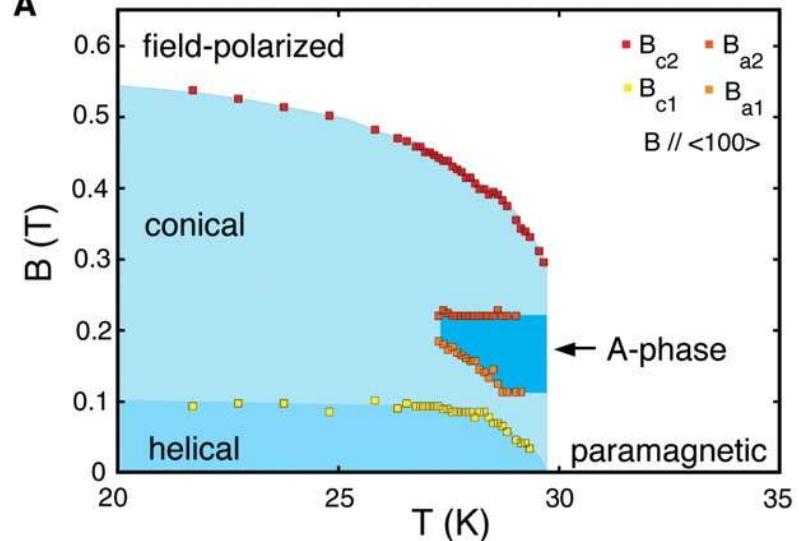
Magnetic state:

- ordered state is helimagnet with $q = 180\text{\AA}$, pinned in (111) direction
- short-range order persists in paramagnetic phase, helical axis depinned



Skyrmions and skyrmion lattices

- even more exotic magnetic states occur in a magnetic field B
- in “A” phase, magnetization vector forms knots, called **skyrmions**, by twisting in two directions
- these skyrmions arrange themselves into regular **skyrmion lattice**



If the critical behavior is classical at any nonzero temperature, why are quantum phase transitions more than an academic problem?

Phase diagrams close to quantum phase transition

Quantum critical point controls **nonzero-temperature** behavior in its vicinity:

Path (a): crossover between classical and quantum critical behavior

Path (b): temperature scaling of quantum critical point

