Project 2:
Simulation of Radioactive Decay

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PH351
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I.   Introduction

This project involved the simulation of radioactive decay. The probability, P, that a certain atom will decay in a small amount of time Δt is:

$$P = \lambda \, \Delta t \tag{1.1}$$

$$\lambda = \ln(2) \, / \, t_{1/2} \tag{1.2}$$

where λ is the time constant and $t_{1/2}$ is the half life. The process of radioactive decay was simulated many times using a stochastic process, and the statistics of the simulation were analyzed.

II.   Procedure

A program was written to compute a time record of surviving atoms R total times. Each run started with N = 1000 radioactive atoms and ran for K = 400 time steps. The time steps existed over an arbitrary time range (0 to t_{max} = 100) where $t_{1/2}$ was set to 0.1 t_{max}. For each time step, all atoms were subjugated to a test consisting of a random number generator made by our lab TA, Rastko Sknepnek. The random number generator (RNG) returned a random value between 0 and 1. P is between 0 and 1, so if the RNG returned a value less than or equal to P, the atom under consideration was assigned to decay within the time step.

The statistics were calculated over all runs as a function of time. Each run required a different seed value. The seed value was simply the number of the present run. These included the average number of surviving atoms, $\langle N(t) \rangle$; the standard deviation, $\sigma(t)$; and the ratio $\sigma(t)^2 / \langle N(t) \rangle$, which is the variance normalized to the average. Note, the average is not the time average, but the average calculated over the runs. Lastly, a histogram of 100 atom bins (10

atoms per bin) and 100 time bins (4 time steps per bin) was calculated and graphed. Time bins were not necessary, but they serve as an average over a short period of time.

III. Results

Firstly considered is the average $\langle N(t) \rangle$, see Figure 3.1. From this plot it can be seen, as expected, the number of surviving atoms decreases exponentially with time. Similarly, it can be said that the rate of decay is proportional to the number of surviving atoms.
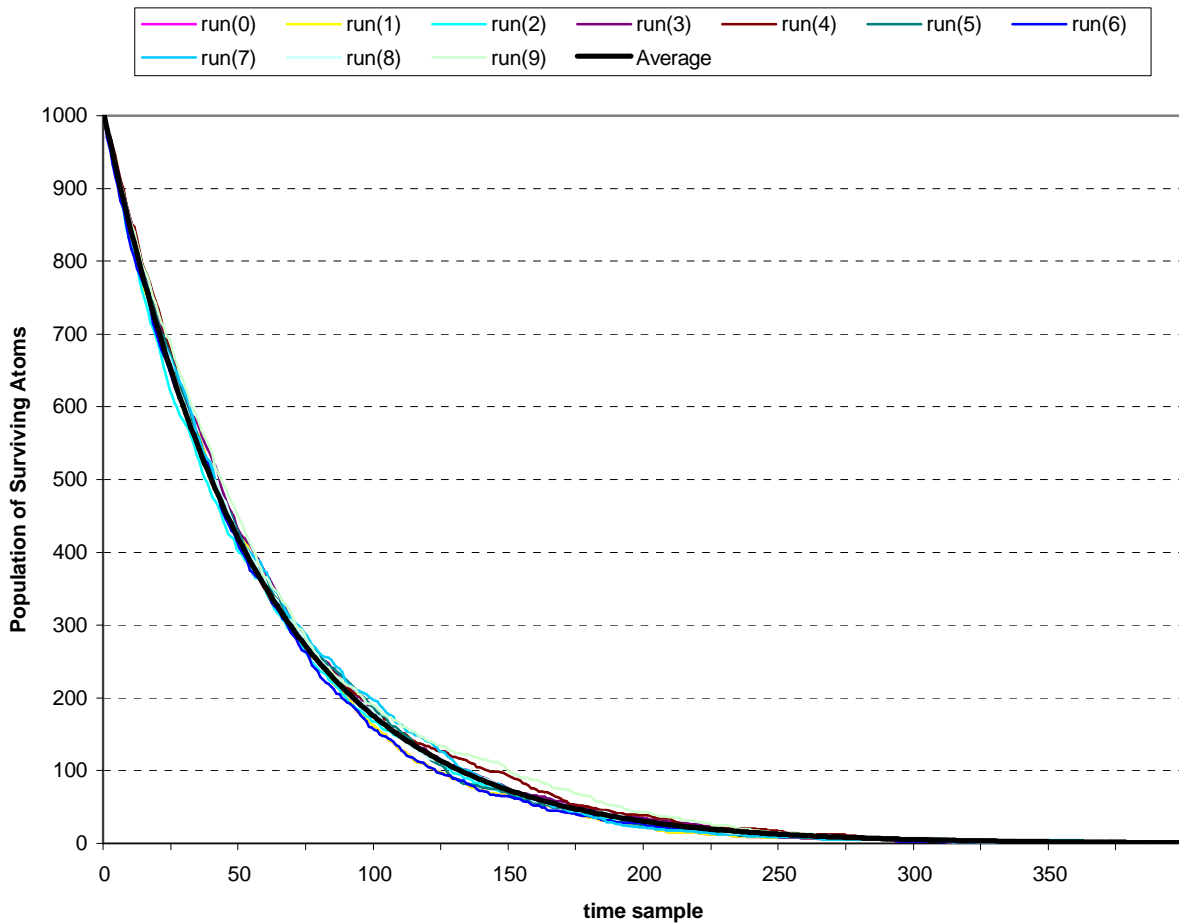

Figure 3.1 Average Number of Surviving Atoms, $\langle N(t) \rangle$.

Secondly, the standard deviation, $\sigma(t)$, is shown in Figure 3.2. Here, the standard

deviation starts from zero. This is expected because at the beginning of all simulations the

radioactive atoms sum up to N or the total number of atoms. As the simulation progresses, the

difference between the runs becomes maximum due to the random nature of the program. As

time continues to progress, the number of surviving atoms converges to zero for all runs.

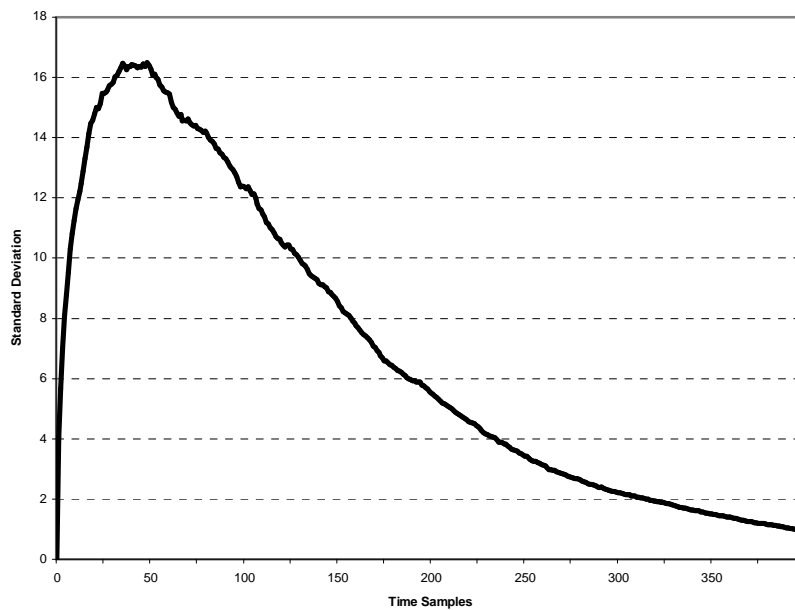Therefore, the standard deviation converges to zero as well.

Figure 3.2 The Standard Deviation as a Function of Time, $\sigma(t)$.

Thirdly, the variance normalized to the average, $\sigma(t)^2 / <N(t)>$, is shown in Figure 3.3.

Here the square of the standard deviation, known as the variance, is shown normalized to the

average number of surviving atoms. At the beginning, it starts from zero for the same reason as

the standard deviation. However, as time progresses the graph does not converge to zero. This

is because $<N(t)>$ becomes the same order as the variance. In other words, $\sigma(t)^2 / <N(t)>$

represents the percentage difference.

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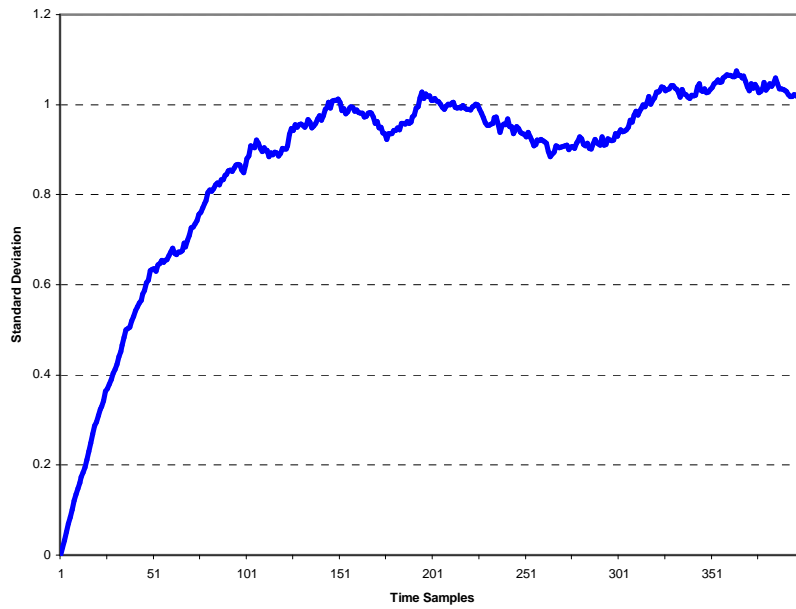**Figure 3.3** The Variance Normalized to the Average, $\sigma(t)^2 / <N(t)>$.
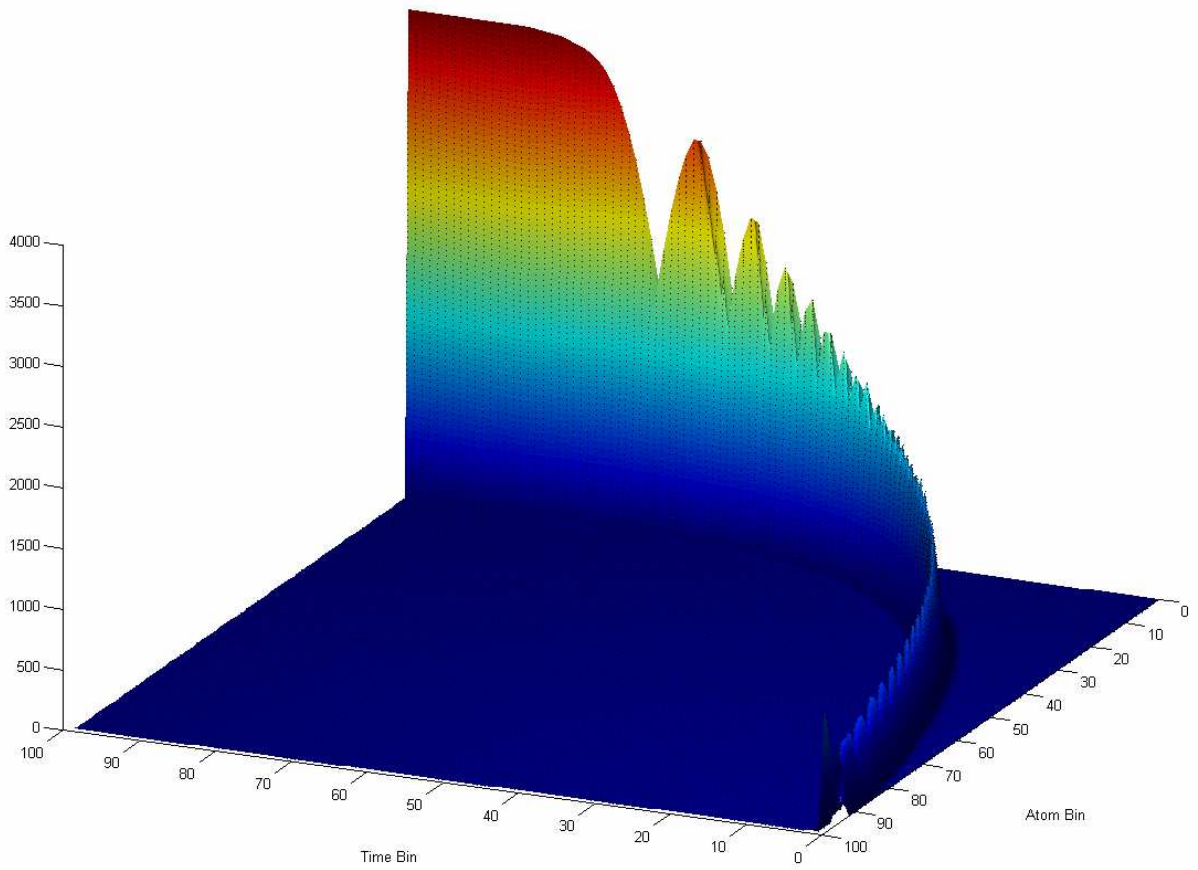

Figure 3.4 Histogram of Surviving Particles as a Function of Time.

Lastly, a histogram of surviving atoms as a function of time is shown in Figure 3.4. When the range of the histogram is far from the boarders of the domain of surviving atoms, the distribution looks Gaussian in nature. This condition lasts for time bins 10 through 90. The width of the curve is dependent on normalized variance.