Physics 5403: Computational Physics – Project 3a

due date: Tuesday, Sep 27, 2022

The solar system

In this project you will explore the motion of planets in the solar system. The planetary motion is governed by Newtonian gravity with the force law

$$\vec{F}_{12} = -\frac{G m_1 m_2}{|\vec{r}_1 - \vec{r}_2|^2}\,\vec{e}_{12}$$

where $\vec{e}_{12}$ is the unit vector in the direction of $\vec{r}_1 - \vec{r}_2$ and $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}. Notice that all planets in the solar system move in (approximately) the same plane; therefore a two-dimensional simulation will be sufficient. In the first part of the project, the motion of a single planet is investigated.

a) Write a program which integrates Newton's equation of motion for a single planet under the influence of the gravitation of the sun (which you can assume not to be moving) from time t_{min} to time t_{max} using a time step τ. Consider a generalized gravitational force $\sim |\vec{r}_1 - \vec{r}_2|^{-\beta}$ ($\beta = 2$ corresponds to normal gravity). Record the position, velocity, kinetic energy, potential energy and total energy as functions of time.

b) Determine a reasonable value for the time step τ. What would be a good initial guess? Why? For one particular planet (e.g., the Earth) and using $\beta = 2$, optimize τ using the requirement of total energy conservation. To this end, calculate the energy change over one orbit and plot it as a function of τ. What type of behavior do you expect? Choose an appropriate way of plotting the result.

c) Check Keplers's third law for all planets with nearly circular orbits (for parameters see below). Think about how to choose the initial conditions to obtain circular orbits.

d) Study what would happen if the force law had an exponent $\beta \neq 2$. Consider the motion of one particular planet for $\beta = 3, 2.5, 2.1$ and 2.01. To see the effect clearly you may want to study a planet having an elliptical orbit with a sizable eccentricity (i.e., Mercury or Pluto). Note: You need to adjust the initial conditions to account for the weaker force.

e) According to general relativity the force law has to be modified to

$$\vec{F}_{12} = -\frac{G m_1 m_2}{|\vec{r}_1 - \vec{r}_2|^2}\,\vec{e}_{12}\left(1 + \frac{\alpha}{|\vec{r}_1 - \vec{r}_2|^2}\right)$$

with $\alpha \approx 1.1 \times 10^{-8}$AU^2. The correction leads to a slow precession (rotation) of the perihelion (the point nearest to the sun) of the mercury orbit. Modify your program to investigate this effect. α is very small, therefore a direct observation of the precession is difficult. What can be done instead? Calculate for several, larger α and extrapolate your result to the correct value.

Planetary data

planet	mass(kg)	semi-major axis (AU)	eccentricity
Mercury	2.4×10^{23}	0.39	0.206
Venus	4.9×10^{24}	0.72	0.007
Earth	6.0×10^{24}	1.00	0.017
Mars	6.6×10^{23}	1.52	0.093
Jupiter	1.9×10^{27}	5.20	0.048
Saturn	5.7×10^{26}	9.54	0.056
Uranus	8.8×10^{25}	19.19	0.046
Neptune	1.0×10^{26}	30.06	0.010
Pluto	1.3×10^{22}	39.26	0.248
Sun	2.0×10^{30}		