Physics 5403: Computational Physics – Project 2

due date: Sep 13, 2022

Radioactive Decay

Radioactive decay is a random process during which atoms decay spontaneously and independently of each other. The probability that a certain atom decays during a small time interval dt is given by

$$dP = \lambda \; dt$$

where λ is the decay constant. This project deals with simulating the stochastic decay process using pseudo-random numbers.

a) Consider an ensemble of N_0 radioactive atoms at time $t = 0$. Derive a differential equation for the time evolution of the average number of surviving atoms $\langle N(t) \rangle$. Solve this differential equation analytically. Relate λ to the half life time $t_{1/2}$.

b) Write a program which simulates the decay process of N_0 individual radioactive atoms over a certain time $t_{max} \gg t_{1/2}$. Define a small time interval δt with $\delta t \ll t_{1/2}$ (Why?), and use a pseudo random number generator to decide whether or not an individual atom decays during a certain slice. Repeat the entire simulation R times with different random number seeds. Calculate the average number $\langle N(t) \rangle$ of surviving particles after each time slice and its standard deviation $\sigma(t)$.

c) Run the simulation for $N_0 = 1000$, $R = 1000$. Think about what values to choose for t_{max} and δt. Plot the number of surviving atoms for the first 10 runs as functions of time together with the theoretically expected values. Plot the average $\langle N(t) \rangle$ and the standard deviation $\sigma(t)$. Plot and discuss the ratio $\sigma^2(t)/\langle N(t) \rangle$.

d) Calculate the distribution function of the number of surviving particles after a time t_{dis}. To do so, define a histogram having N_{bin} bins of width w_{bin}. Find reasonable values for these two parameters. Plot the histogram.