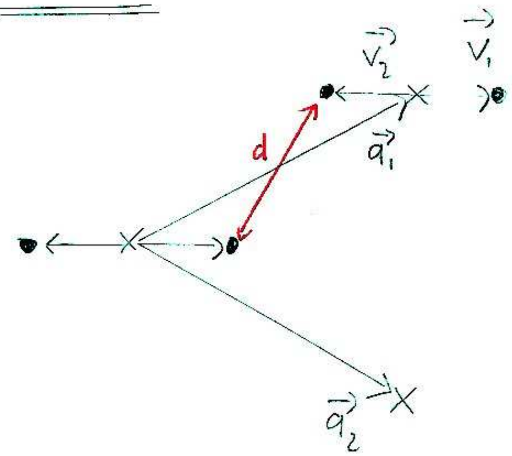
Physics 481: Condensed Matter Physics - Solutions 1

Problem 1.1.

a)



– distance between points of basis pair.

$$|\vec{v}_1 - \vec{v}_2| = \frac{\sqrt{3}}{3} a$$

– distance

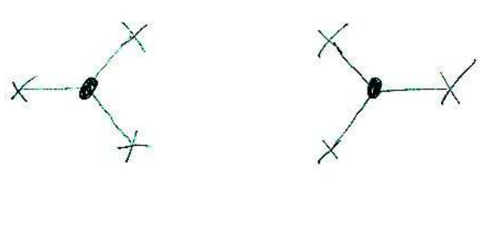
$$d = |-\vec{v}_1 + \vec{a}_1 + \vec{v}_2| = \left| a \begin{pmatrix} -\frac{\sqrt{3}}{6} + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} \\ \frac{1}{2} \end{pmatrix} \right|$$

$$= \left| a \begin{pmatrix} \sqrt{3}/6 \\ 1/2 \end{pmatrix} \right| = a\sqrt{\frac{3}{36} + \frac{1}{4}} = a\sqrt{4/12}$$

$$= a\,\frac{\sqrt{3}}{3}$$

all other distances are equivalent

b)



rotation by $60°$

Problem 1.2

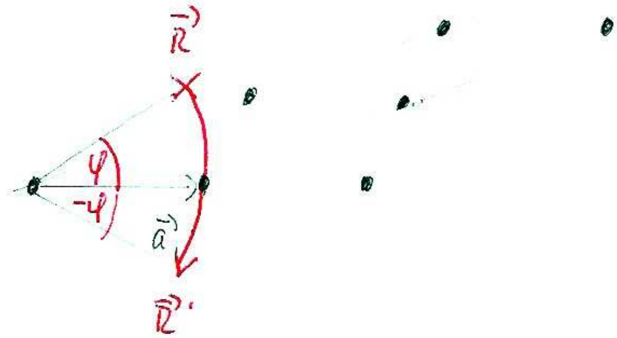
$$\underline{M} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}$$

$$= \begin{pmatrix} \cos\varphi & \sin\varphi \\ \sin\varphi & -\cos\varphi \end{pmatrix} \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\varphi - \sin^2\varphi & 2\sin\varphi\cos\varphi \\ 2\sin\varphi\cos\varphi & -\cos^2\varphi + \sin^2\varphi \end{pmatrix}$$

$$= \begin{pmatrix} \cos(2\varphi) & \sin(2\varphi) \\ \sin(2\varphi) & -\cos(2\varphi) \end{pmatrix}$$

Problem 1.3



basis vectors $\quad \vec{a}_1 = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad \vec{a}_2 = \begin{pmatrix} A \\ B \end{pmatrix}$

$$\vec{R} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} a\cos\varphi \\ a\sin\varphi \end{pmatrix} \overset{!}{=} n_1 \vec{a}_1 + n_2 \vec{a}_2$$

$$\vec{R}' = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} a\cos\varphi \\ -a\sin\varphi \end{pmatrix} \overset{!}{=} m_1 \vec{a}_1 + m_2 \vec{a}_2$$

$$a\cos\varphi = n_1 a + n_2 A = m_1 a + m_2 A \qquad (\mathrm{I})$$

$$a\sin\varphi = n_2 B = -m_2 B \qquad (\mathrm{II})$$

(II): $\quad m_2 = -n_2$

(I):

$$a\cos\varphi = n_1 a + n_2 A$$
$$a\cos\varphi = m_1 a - n_2 A$$

$$\cos\varphi = \frac{1}{2}(n_1 + m_1)$$

$\cos\varphi$ is half-integer

$n_1 + m_1 =$	
1	$\varphi = 0$
$\frac{1}{2}$	$\varphi = 60°$
0	$\varphi = 90°$
$-\frac{1}{2}$	$\varphi = 120°$
-1	$\varphi = 180°$