Physics 481: Condensed Matter Physics - Homework 8

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due date: March 18, 2011

Problem 1: Damped oscillations (10 points)

In a linear chain of lattice spacing a, particles of mass m are connected by nearest-neighbor springs of spring constant K. In addition to the elastic forces, each particle is subjected to a damping force $F_D = -\Gamma \dot{u}_n$, where u_n is the displacement of the nth particle from the equilibrium position. How does the damping change the frequencies $\omega(k)$, and what is the relaxation time of the modes? Assume $(\Gamma/m)^2 \ll K/m$ and discuss the limiting cases $k \approx 0$ and $k \approx \pi/a$.

Problem 2: Creation and destruction operators (15 points)

Consider a quantum-mechanical harmonic oscillator with Hamiltonian

$$H = \frac{p^2}{2M} + \frac{1}{2} M\omega^2 x^2$$

where M is the mass, ω is the frequency and p and x and the momentum and position operators fulfilling the commutator $[p, x] = \hbar/i$. The destruction and creation operators are defined by

$$\begin{aligned} a &= \sqrt{\frac{M\omega}{2\hbar}} x + \frac{i}{\sqrt{2M\omega\hbar}} p \\ a^\dagger &= \sqrt{\frac{M\omega}{2\hbar}} x - \frac{i}{\sqrt{2M\omega\hbar}} p \end{aligned}$$

a) Calculate the commutator $[a, a^\dagger]$.
b) Show that the Hamiltonian can be written as $H = \hbar\omega(a^\dagger a + 1/2)$.
c) $|n\rangle$ denotes the *normalized* eigenstate with energy $E_n = \hbar\omega(n + 1/2)$. Show that $a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$ and $a|n\rangle = \sqrt{n}\,|n-1\rangle$.

Problem 3: Low-temperature specific heat in d dimensions and for nonlinear dispersion laws (Ashcroft-Mermin problem 23.2, 15 points)

Consider small lattice vibrations in a d-dimensional crystal in harmonic approximation.

a) For the Debye model, i.e. a linear dispersion $\omega = c|\mathbf{k}|$ of all phonon modes, calculate the phonon density of states and show that it varies as ω^{d-1}. What is the Debye frequency?
b) Determine the phonon contribution to low-temperature specific heat.
c) Investigate what would happen for a nonlinear phonon dispersion $\omega \sim |\mathbf{k}|^\nu$ (anomalous sound). Show that the low-temperature specific heat would vanish as $T^{d/\nu}$ in d dimensions.