9.1. $$d\underline{U} = T\,dS + \sigma\,dA$$

a) $$H = \underline{U} - \sigma A$$

$$dH = d\underline{U} - d(\sigma A) = T\,dS - A\,d\sigma$$

b) $$F = \underline{U} - TS$$

$$dF = d\underline{U} - d(TS) = -S\,dT + \sigma\,dA$$

c) $$G = F - \sigma A$$

$$dG = dF - d(\sigma A) = -S\,dT - A\,d\sigma$$

9.2

$$\left(\frac{\partial \sigma}{\partial S}\right)_A = \left(\frac{\partial T}{\partial A}\right)_S \qquad (\text{from } d\underline{U})$$

$$\left(\frac{\partial A}{\partial S}\right)_\sigma = -\left(\frac{\partial T}{\partial \sigma}\right)_S \qquad (\text{from } dH)$$

$$\left(\frac{\partial S}{\partial A}\right)_T = -\left(\frac{\partial \sigma}{\partial T}\right)_A \qquad (\text{from } dF)$$

$$\left(\frac{\partial S}{\partial \sigma}\right)_T = \left(\frac{\partial A}{\partial T}\right)_\sigma \qquad (\text{from } dG)$$

9.3 a) $$d\underline{u} = T\,dS - p\,dV$$

$$dS = \frac{1}{T}\,d\underline{u} + \frac{p}{T}\,dV$$

use $$d\underline{u} = \left(\frac{\partial u}{\partial T}\right)_V dT + \left(\frac{\partial u}{\partial V}\right)_T dV$$

$$dS = \frac{1}{T}\left(\frac{\partial u}{\partial T}\right)_V dT + \left[\frac{p}{T} + \frac{1}{T}\left(\frac{\partial u}{\partial V}\right)_T\right] dV$$

- mixed 2nd derivatives of S must be equal

$$\frac{1}{T}\frac{\partial}{\partial V}\left[\left(\frac{\partial u}{\partial T}\right)_V\right]_T = \frac{\partial}{\partial T}\left[\frac{p}{T} + \frac{1}{T}\left(\frac{\partial u}{\partial V}\right)_T\right]_V$$

$$\cancel{\frac{1}{T}\frac{\partial^2 u}{\partial V \partial T}} = -\frac{1}{T^2}\left[p + \left(\frac{\partial u}{\partial V}\right)_T\right] + \frac{1}{T}\left[\cancel{\frac{\partial^2 u}{\partial T \partial V}} + \left(\frac{\partial p}{\partial T}\right)_V\right]$$

$$\left(\frac{\partial u}{\partial V}\right)_T = -p + T\left(\frac{\partial p}{\partial T}\right)_V$$

b) $$pV = Nk_BT \qquad p = \frac{Nk_BT}{V}$$

$$\left(\frac{\partial u}{\partial V}\right)_T = -p + T\,\frac{Nk_B}{V} = -p + p = 0$$