1 a) $$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \qquad V = \frac{Nk_BT}{p}$$

$$= +\frac{1}{V}\frac{Nk_BT}{p^2} = \frac{1}{p}$$

b) on adiabatic curve $TV^{2/3} = \text{const}$

using $pV = Nk_BT$ $\Rightarrow$ $pV^{5/3} = \text{const}$

$$p^{3/5}V = A$$

$$\kappa_S = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_S$$

$$= -\frac{1}{V}\frac{\partial}{\partial p}\frac{A}{p^{3/5}} = \frac{3}{5}\frac{A}{p^{8/5}} = \frac{3}{5}\frac{1}{p}$$

2a) $$dU = TdS - pdV$$

$$dS = \frac{dU}{T} + \frac{p}{T}dV = \frac{3}{2}Nk_B\frac{dT}{T} + Nk_B\frac{dV}{V}$$

$$S - S_0 = \frac{3}{2}Nk_B\ln(T/T_0) + Nk_B\ln(V/V_0)$$

b) for $T \to 0$, $S \to -\infty$

unphysical, ideal gas model breaks down at low T

3) to get equation for adiabatic curve

$$dU = \delta W \qquad \frac{3}{2} \cancel{Nk_B}\, dT = -p\, dV = -\frac{\cancel{Nk_B} T}{V} dV$$

$$\frac{3}{2} \frac{dT}{T} = -\frac{dV}{V} \Rightarrow \frac{3}{2} \ln T/T_0 = -\ln \frac{V}{V_0}$$

$$T/T_0 \left(V/V_0\right)^{2/3} = 1 \qquad T V^{2/3} = \text{Const}$$

a) $$T_A V_A^{2/3} = T_B V_B^{2/3} \qquad T_B = T_A \left(\frac{V_A}{V_B}\right)^{2/3}$$

$$T_C V_B^{2/3} = T_D V_A^{2/3} \qquad T_D = T_C \left(\frac{V_B}{V_A}\right)^{2/3}$$

b) $$Q_{BC} = C_V (T_C - T_B)$$

$$Q_{DA} = C_V (T_A - T_D)$$

c) $$\Delta U = 0 = W + Q_{BC} + Q_{DA}$$

$$W = -Q_{BC} - Q_{DA} = C_V \left(T_B - T_C + T_D - T_A\right)$$

d) $$\eta = \frac{|W|}{Q_{BC}} = \frac{|T_B - T_C + T_D - T_A|}{T_C - T_B} = 1 + \frac{T_A - T_D}{T_C - T_B}$$

e) $$\eta = 1 + \frac{T_A - T_D}{T_D \left(\frac{V_A}{V_B}\right)^{2/3} - T_A \left(\frac{V_A}{V_B}\right)^{2/3}} = 1 - \frac{1}{r^{2/3}}$$