4.1 a) $\sigma = \pi (a_1 + a_2)^2 = 4\pi a^2$ $\quad a = 0.53 \text{ Å}$

$\sigma = 3.53 \times 10^{-20} \text{ m}^2$

b) speed $\langle v \rangle = \sqrt{\frac{8}{\pi} \frac{k_B T}{m}} = 797 \text{ m/s}$

$m = 1.66 \times 10^{-27} \text{ kg}$

$v_r \approx \sqrt{2} \langle v \rangle = 1124 \text{ m/s}$

$\tau = \frac{1}{n \sigma v_r} = 5.04 \times 10^{7} \text{ s}$

c) $\lambda = \frac{1}{\sqrt{2} n \sigma} = 4.02 \times 10^{10} \text{ m}$

4.2 a) $\frac{dN}{dt} = -\phi A = -\frac{1}{4}\langle v\rangle n A = -\frac{1}{4}\langle v\rangle \frac{N}{V} A$

$pV = Nk_BT \qquad p = \frac{Nk_BT}{V}$

$\frac{dp}{dt} = -\frac{1}{4}\frac{A}{V}\langle v\rangle p \qquad \langle v\rangle = \sqrt{\frac{8}{\pi}\frac{k_BT}{m}}$

b) $p = p_0 e^{-\frac{1}{4}\frac{A}{V}\langle v\rangle t}$

$= p_0 e^{-\frac{A}{V}t\sqrt{\frac{k_BT}{2\pi m}}}$

4.3 a) $du_a = 2x\,dx + dy$

$\frac{\partial}{\partial y}(2x) = \frac{\partial}{\partial x}(1) = 0$ du_a exact

$du_b = y\,dx$

$\frac{\partial}{\partial y}(y) \neq \frac{\partial}{\partial x}(0)$ du_b not exact

b) $u_a(x,y) = x^2 + y + \text{const}$

c) $\underline{du_a}$

$I_1 = \int_0^2 dx\, 2x + \int_0^2 dy = 4 + 2 = 6$

$I_2 = \int_0^2 dy + \int_0^2 dx\, 2x = 2 + 6 = 6$

$u_a(2,2) - u_a(0,0) = 6$

$\underline{du_b}$

$I_1 = \int_0^2 0\,dx + \int_0^2 0\,dy = 0$

$I_2 = \int_0^2 0\,dy + \int_0^2 2\,dx = 4$

integral over du_b depends on path