

3.1)

Maxwell distribution

$$g(\vec{v}) = \left( \frac{m}{2\pi k_B T} \right)^{3/2} e^{-\frac{1}{2} m \vec{v}^2 / k_B T}$$

$$\langle \vec{v}^2 \rangle = 3 \frac{k_B T}{m}$$

$$\sqrt{\langle \vec{v}^2 \rangle} = \sqrt{3 \frac{k_B T}{m}} = v_{rms}$$

$$a) \quad m = 3.32 \times 10^{-27} \text{ kg} \quad T = 300 \text{ K}$$

$$v_{rms} = 1934 \text{ m/s}$$

$$b) \quad m = 6.65 \times 10^{-27} \text{ kg}$$

$$v_{rms} = 1367 \text{ m/s}$$

$$c) \quad m = 7.31 \times 10^{-26} \text{ kg}$$

$$v_{rms} = 412 \text{ m/s}$$

3.2)

$$a) \quad g(\vec{v}) = \frac{m}{2\pi k_B T} e^{-\frac{m}{2}(v_x^2 + v_y^2)/k_B T}$$

$$b) \quad \langle \vec{v}^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle = 2 \frac{k_B T}{m}$$

$$v_{rms} = \langle \vec{v}^2 \rangle^{1/2} = \sqrt{2 \frac{k_B T}{m}}$$

$$c) \quad \bar{P} = \int_{v_{min}}^{\infty} dv v \int_0^{2\pi} d\varphi \frac{m}{2\pi k_B T} e^{-\frac{m}{2} v^2 / k_B T}$$

$$\text{with } v_{min} = 2 \sqrt{2 \frac{k_B T}{m}}$$

$$\bar{P} = \frac{m}{k_B T} \int_{v_{min}}^{\infty} dv v e^{-\frac{m}{2} v^2 / k_B T}$$

$$u = \frac{1}{2} v^2$$

$$u_{min} = 4 \frac{k_B T}{m}$$

$$\bar{P} = \frac{m}{k_B T} \int_{u_{min}}^{\infty} du e^{-m u / k_B T}$$

$$\bar{P} = e^{-m u_{min} / k_B T} = e^{-4}$$

$$3.3 a) \quad \langle \omega \rangle = \omega_0 \left( 1 - \frac{\langle v_x \rangle}{c} \right) = \omega_0 \quad \text{by symmetry}$$

$$\begin{aligned} b) \quad \langle \omega^2 \rangle &= \omega_0^2 \left\langle \left( 1 - \frac{v_x}{c} \right)^2 \right\rangle \\ &= \omega_0^2 \left( 1 - 2 \frac{\langle v_x \rangle}{c} + \frac{\langle v_x^2 \rangle}{c^2} \right) \\ &= \omega_0^2 \left( 1 + \frac{1}{c^2} \frac{k_B T}{m} \right) \end{aligned}$$

$$\sigma_\omega^2 = \langle \omega^2 \rangle - \langle \omega \rangle^2 = \frac{\omega_0^2}{c^2} \frac{k_B T}{m}$$