10.1) a) $dU = \bar{T}\,dS + f\,dL$

b) $F = U - \bar{T}S$

$dF = -S\,d\bar{T} + f\,dL$

$$\left(\frac{\partial S}{\partial L}\right)_{\bar{T}} = -\left(\frac{\partial f}{\partial \bar{T}}\right)_L$$

c) $\Delta S = \Delta S_1 + \Delta S_2$

$(\bar{T}_0 \to \bar{T} \text{ @ } L_0)$ $(L_0 \to L \text{ @ } T)$

$$\Delta S_1 = \int dS = \int \frac{dU}{\bar{T}} = \int_{\bar{T}_0}^{\bar{T}} \left(\frac{\partial U}{\partial \bar{T}}\right|_L \frac{d\bar{T}}{\bar{T}}$$

$$= b\int_{\bar{T}_0}^{\bar{T}} d\bar{T} = b(\bar{T} - \bar{T}_0)$$

$$\Delta S_2 = \int_{L_0}^{L} \left(\frac{\partial S}{\partial L}\right)_{\bar{T}} dL = -\int_{L_0}^{L} 2a\bar{T}(L' - L_0)\,dL'$$

$$= -a\bar{T}(L - L_0)^2$$

$$\Delta S = b(\bar{T} - \bar{T}_0) - a\bar{T}(L - L_0)^2$$

d)

$$S_i = S_0 + b(\bar{T}_i - \bar{T}_0) - a\bar{T}_i(L_i - L_0)^2$$

$$S_f = S_0 + b(\bar{T}_f - \bar{T}_0) - a\bar{T}_f(L_f - L_0)^2$$

adiabatic $S_i = S_f$

$$\bar{T}_f = \bar{T}_i \frac{b - a(L_i - L_0)^2}{b - a(L_f - L_0)^2}$$

10.2) a) move wire to the right by dx

$\Rightarrow dA = 2\ell\, dx$

$\sigma\, dA = f\, dx \qquad f = 2\ell\sigma$ (to left)

b) $dU = \bar{T}\, dS + 2\ell\sigma\, dx$

c) $W = \int_0^x f\, dx' = 2\ell\sigma \int_0^x dx' = 2\ell\sigma x$

d) $\Delta U = \int_{x'=0}^{x} dU = \int_{x'=0}^{x} \left(\frac{\partial U}{\partial x}\right)_{\bar{T}=\bar{T}_0} dx$

$$\left(\frac{\partial U}{\partial x}\right)_{\bar{T}} = \bar{T}\left(\frac{\partial S}{\partial x}\right)_{\bar{T}} + f = \bar{T}\left(\frac{\partial S}{\partial x}\right)_{\bar{T}} + 2\sigma\ell$$

Maxwell relation for $\bar{F}$: $d\bar{F} = -S\, d\bar{T} + 2\gamma\ell\, dx$

$$\left(\frac{\partial S}{\partial x}\right)_{\bar{T}} = -\left(\frac{\partial}{\partial \bar{T}}(2\sigma\ell)\right)_x = -2\ell(-a) = 2\ell a$$

$$\left(\frac{\partial U}{\partial x}\right)_{\bar{T}} = 2\ell a\bar{T} + 2\sigma\ell = 2\sigma_0\ell$$

$$U(x) + U(0) = 2\sigma_0\ell x$$

10.3) $\Delta S = \int dS = \int_0^{B_0} \left(\frac{\partial S}{\partial B}\right)_T dB$ [assume $B \propto H$]

$$F = -S\,d\bar{T} - m\,dB$$

$$\left(\frac{\partial S}{\partial B}\right)_T = +\left(\frac{\partial m}{\partial T}\right)_B = -\frac{A}{(T-T_w)^2}B$$

$$\Delta S = \int_0^{B_0} -\frac{A}{(T-T_w)^2} B\,dB = -\frac{1}{2}\frac{AB^2}{(T-T_w)^2}$$