1.1 a) $1/20 + 1/10 + 1/10 + 3/20 + 3/10 + 3/10$

$= 5/20 + 15/20 = 1$ normalized

b) $p_x(0) = p_{xy}(0,5) + p_{xy}(0,7) + p_{xy}(0,9) = 1/4$

$p_x(2) = p_{xy}(2,5) + p_{xy}(2,7) + p_{xy}(2,9) = 3/4$

c) $p_y(5) = p_{xy}(0,5) + p_{xy}(2,5) = 1/5$

$p_y(7) = p_{xy}(0,7) + p_{xy}(2,7) = 2/5$

$p_y(9) = p_{xy}(0,9) + p_{xy}(2,9) = 2/5$

d) $p_x(0|y=5) = \frac{p_{xy}(0,5)}{p_y(5)} = \frac{1/20}{1/5} = \frac{1}{4}$

$p_x(0|y=7) = \frac{p_{xy}(0,7)}{p_y(7)} = \frac{1/10}{2/5} = \frac{1}{4}$

$p_x(0|y=9) = \frac{p_{xy}(0,9)}{p_y(9)} = \frac{1/10}{2/5} = \frac{1}{4}$

e) - $p_x(0|y)$ does not depend on y $\Rightarrow$ statistically independent

- $p_{xy}(x,y) = p_x(x)\, p_y(y)$ $\Rightarrow$ statistically independent

1.2. Gaussian distribution

a) $C = \frac{1}{\sqrt{2\pi a^2}}$

b) $\langle x \rangle = b$ (follows directly after substitution $x - b = x'$)

$x_m = b$

$x_p = b$

c) $\sigma_x^2 = a^2$

$\langle x^2 \rangle = \sigma^2 + \langle x \rangle^2 = a^2 + b^2$

1.3 a) $P(4) = \binom{10}{4}\left(\frac{1}{2}\right)^{10} = \frac{210}{1024} = 0.2051$

$P(5) = \binom{10}{5}\left(\frac{1}{2}\right)^{10} = \frac{252}{1024} = 0.2461$

$P(4)/P(5) = 5/6$

b) $P(40) = \binom{100}{40}\left(\frac{1}{2}\right)^{100} = 0.01084$

$P(50) = \binom{100}{50}\left(\frac{1}{2}\right)^{100} = 0.07959$

$P(40)/P(50) = 0.1362$

c) $P(400) = \binom{1000}{400}\left(\frac{1}{2}\right)^{1000} = 4.634 \times 10^{-11}$

$P(500) = \binom{1000}{500}\left(\frac{1}{2}\right)^{1000} = 0.02522$

$P(400)/P(500) = 1.837 \times 10^{-9}$

$\left.\right\}$ using Wolfram Alpha

<u>alternatively</u>

$$\ln P(400) = \ln\left(\frac{1000!}{40!\,60!}\right) + \ln\left(\left(\frac{1}{2}\right)^{1000}\right)$$

$$= \ln(1000!) - \ln(40!) - \ln(60!) - 1000\ln(2)$$

$$= \frac{1}{2}\ln(2\pi\times1000) + 1000\ln(1000) - 1000$$
$$- \frac{1}{2}\ln(2\pi\times400) - 400\ln(400) + 400$$
$$- \frac{1}{2}\ln(2\pi\times600) - 600\ln(600) + 600$$
$$- 1000\ln 2$$

$$= -23.795$$

and analogously for the other terms