Physics 4311: Thermal Physics - Homework 8

due date: Tuesday, March 19, 2024, please upload your solution as a pdf on Canvas

Problem 1: Compressibilities of the ideal gas (10 points)

An ideal gas obeys the equation of state $pV = Nk_BT$. The internal energy is $U = (3/2)Nk_BT$. Starting from these relations, compute the following quantities

a) isothermal compressibility κ_T

b) adiabatic compressibility κ_S

Problem 2: Entropy of the ideal gas (10 points)

The equation of state of an ideal gas is $pV = Nk_BT$ with p being pressure, V volume, N the number of particles, k_B the Boltzmann constant, and T the temperature. The internal energy is given by $U = (3/2)Nk_BT$.

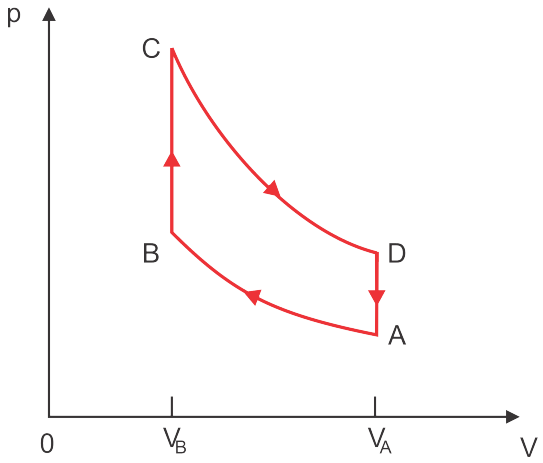
a) Start from the first law, $dU = T\,dS - p\,dV$, and derive an expression for the entropy of the ideal gas as a function of T and V.

b) Determine the behavior of S for $T \to 0$. What does the result mean?

Problem 3: Otto cycle (20 points)

The Otto cycle shown in the picture is an idealized version of the process taking place in a gasoline internal combustion engine.

Consider an Otto cycle using a (monoatomic) classical ideal gas of N atoms as working medium. The cycle consists of an adiabatic compression (A $\to$ B), an isochoric heating (B $\to$ C), an adiabatic expansion (C $\to$ D), and an isochoric cooling (D $\to$ A).



a) Use the adiabatic temperature-volume relation to express the temperature T_B in terms of T_A, V_A, and V_B. Also express the temperature T_C in terms of T_D, V_A, and V_B.

b) Find the heat Q_{BC} absorbed in process B $\to$ C and the heat Q_{DA} released in the process D $\to$ A in terms of the temperatures T_A, T_B, T_C, and T_D.

c) Find the work W done by the engine during one cycle. (Use the 1st law!)

d) The efficiency of the cycle is defined as $\eta = |W|/Q_{BC}$. Compute the efficiency in terms of the temperatures T_A, T_B, T_C, and T_D.

e) Show that the efficiency depends on the compression ratio $r = V_A/V_B$ only. Express your result for the efficiency in terms of r.