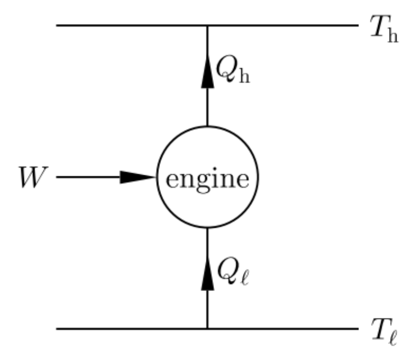
Physics 4311: Thermal Physics - Homework 6

due date: Tuesday, March 5, 2024, please upload your solution as a pdf on Canvas

Problem 1: Heat pump (16 points)

A heat pump is a device that uses work to transport heat from a low-temperature reservoir to a high-temperature reservoir (see figure on the right). Its efficiency can be defined as the ratio of the heat deposited into the high-temperature reservoir and the work done on the system, $\eta = |Q_h|/W$.



a) Consider a heat pump consisting of a Carnot engine running backwards. Using the results for the ideal-gas based Carnot engine discussed in class, find the efficiency of this "ideal" heat pump in terms of T_h and T_l.

b) Is the efficiency larger or smaller than unity? Explain what the result means.

c) Heat pumps can be used to heat buildings. What is the efficiency of an ideal (Carnot) heat pump that takes heat from the outside air at 40 °F and transports it to the inside of the building which is at 70 °F?

d) What is the efficiency of the heat pump in part c) if the outside air is at a temperature of 0 °F?

Problem 2: Carnot process for a paramagnetic substance (24 points)

Consider a paramagnetic substance whose equation of state reads $M = \alpha H/T$ where T is the temperature, M is the magnetization, H is the magnetic field, and α is a material specific constant. The internal energy is given by $U = CT$ where the specific heat C is a constant. The work differential for a paramagnet is $\delta W = HdM$

a) Consider an isothermal change of magnetization from M_1 to M_2. Compute the work W_{12} done on the system and the heat Q_{12} absorbed by the system.

b) Consider an adiabatic change of magnetization from M_2 to M_3. Find the adiabatic $H - M$ curves by starting from $\delta Q = 0$ and integrating the resulting differential equation.

c) Sketch a Carnot cycle, consisting of two isothermal changes in M and two adiabatic changes in M in the $M - H$ plane.

d) Compute the total work during the cycle and the heat absorbed during the four segments of the cycle.

e) Explicitly calculate the efficiency.

Hint: The derivation is analogous to that of the Carnot cycle for the ideal gas, but using the equation of state $M = \alpha H/T$ instead of the ideal gas law. This leads to changes in some of the mathematical expressions.