Physics 4311: Thermal Physics - Homework 12

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due date: Tuesday, April 23, 2024, please upload your solution as a pdf on Canvas

Problem 1: Generalized equipartition theorem (10 points)

Consider a classical degree of freedom q that makes a contribution to the Hamiltonian of the form $\frac{1}{2}A|q|^n$ where n and A are positive constants. Find the average internal energy stored in this degree of freedom as a function of temperature.

Problem 2: Ideal gas in rotating cylinder (15 points)

Consider a non-relativistic classical ideal gas of N particles (mass m) at temperature T in a cylindrical vessel of radius R and height H. The cylinder is rotating around its vertical axis with angular velocity ω.

a) Compute the partition function [Hint: Work in a rotating reference frame and neglect the Coriolis force.]

b) calculate the internal energy and the specific heat of the gas as functions of temperature.

c) Calculate how the particle density $n(r)$ changes with the distance r from the rotation axis. (Hint: the particle density $n(r)$ is a reduced probability density of the phase space density $\rho(\vec{r}, \vec{p})$.)

Problem 3: Ultra-relativistic classical ideal gas (15 points)

Consider a gas of N non-interacting, indistinguishable, classical particles at temperature T in a cubic box of linear size L. The energy-momentum relation is ultra-relativistic, $E = \mathrm{c}|\vec{\mathrm{p}}_\mathrm{i}|$, where c is the speed of light.

a) Calculate the partition function and the free energy of the gas.

b) Calculate the pressure as function of N, T, and V.

c) Find the internal energy U and the specific heat C_V at constant volume.

d) Also determine the specific heat at constant pressure, and compare the ratio of to that of the nonrelativistic case.