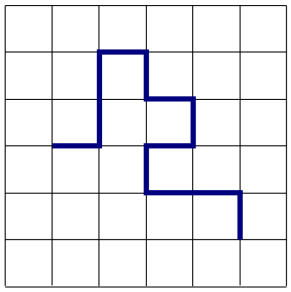
Physics 4311: Thermal Physics - Homework 11

due date: Tuesday, April 16, 2024, please upload your solution as a pdf on Canvas

Problem 1: Polymer on a lattice (20 points)

A polymer can be modeled as a path of $N+1$ identical segments on a square lattice (see picture) connected by N independent joints (lattice sites). At each of the N joints, the polymer can either go straight, or it can bend by 90 degrees left or right. (Different segments of the polymer do not interact with each other, i.e., the path can intersect itself.) A straight joint has zero energy while a right-angle joint has positive energy ϵ. Assume that one end of the polymer is at a fixed position.



a) Find the partition function of this polymer as function of temperature T, the joint energy ϵ, and the number of joints N.

b) Calculate the internal energy U of the polymer.

c) Find the average number $\langle N_{st} \rangle$ of straight joints as a function of temperature T, ϵ, and N.

d) How does $\langle N_{st} \rangle$ behave for $T \to 0$ and $T \to \infty$?

Problem 2: System of 3-level atoms (20 points)

Consider a system of N independent atoms, each having three states $|1\rangle, |2\rangle, |3\rangle$ with energies $\epsilon_1 = \epsilon_2 = 0$, $\epsilon_3 = \epsilon > 0$. The system is in equilibrium with a heat bath at temperature T .

a) Compute the probabilities p_1, p_2, p_3 for a particular atom being in state $|1\rangle, |2\rangle$ or $|3\rangle$.

b) Calculate the partition functions for a single atom and for the N-atom system.

c) Calculate the Helmholtz free energy, the internal energy and the heat capacity.

d) Calculate the entropy and discuss its behavior for $T \to 0$. Where does the excess entropy come from?