Physics 4311: Thermal Physics - Homework 10

due date: Tuesday, April 9, 2024, please upload your solution as a pdf on Canvas

Problem 1: Elastic rod (16 points)

An elastic rod has an unstretched length L_0. If it is stretched to length L, the tension force is given by $f = aT^2(L - L_0)$ where T is the temperature and a is a constant. The heat capacity C_L at constant length is given by $C_L = bT$ when $L = L_0$. Here, b is another constant.

a) Write down the first law in the appropriate variables.

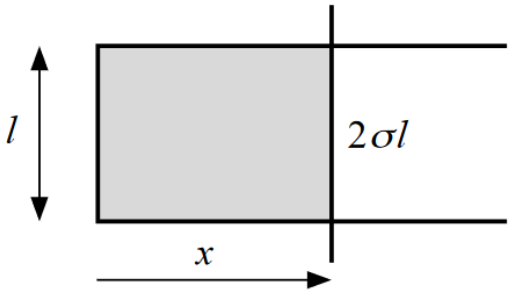
b) Using a Maxwell relation, find $(\partial S/\partial L)_T$

c) Compute the change in entropy as the rod is stretched from length L_0 at temperature T_0 to length L at temperature T. (Hint: It may be useful to split the process into one at constant $L = L_0$ and one at constant temperature.)

d) The rod is stretched adiabatically (thermally isolated) from some initial length L_i at temperature T_i to a final length L_f. Find the final temperature T_f.

Problem 2: Soap film (16 points)

The figure illustrates a soap film (shown in gray) supported by a wire frame. The right cross wire can slide left or right without friction on the rest of the frame. The soap has a positive surface tension σ; its temperature dependence is given by $\sigma = \sigma_0 - aT$. where σ_0 and a are constants. Note that the film has two surfaces (top and bottom).



a) Derive an expression for the force of the film on the cross wire. What is the direction of this force?

b) Write down the first law, i.e., a relation expressing the change dU in internal energy of the film in terms of the heat TdS absorbed by it and the work done on it when the distance x is changed by an amount dx.

c) Calculate the work done on the film in order to stretch it at a constant temperature T_0 from a length 0 to a length x.

d) Calculate the change in internal energy $\Delta U = U(x) - U(0)$ of the film when it is stretched at a constant temperature T_0 from a length 0 to a length x. [Hint: Use the Maxwell relation arising from the Helmholtz free energy to deal with the heat term in dU).

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Problem 3: Magnet (8 points)

The magnetic susceptibility of some magnetic solid is given by the Curie-Weiss law, $\chi = A/(T - T_W)$ where A and the Weiss temperature T_W are constants. How much does the entropy of this solid change if the magnetic field H is increased from 0 to H_0 at fixed temperature T?