Physics 4311: Thermal Physics - Homework 1

due date: Tuesday, Jan 30, 2024, please upload your solution as a pdf on Canvas

Problem 1: Joint probabilities (13 points)

The random variables x and y are jointly distributed. x can take values 0 or 2, whereas y can take the values 5, 7, or 9. The joint probabilities are given by $p_{xy}(0,5) = 1/20$, $p_{xy}(0,7) = 1/10$, $p_{xy}(0,9) = 1/10$, $p_{xy}(2,5) = 3/20$, $p_{xy}(2,7) = 3/10$, $p_{xy}(2,9) = 3/10$.

a) Check that p_{xy} is properly normalized.

b) Compute the reduced probabilities $p_x(0)$, $p_x(2)$.

c) Compute the reduced probabilities $p_y(5)$, $p_y(7)$, $p_y(9)$

d) Compute the conditional probabilities $p_x(0|y = 5)$, $p_x(0|y = 7)$, and $p_x(0|y = 9)$.

e) Determine whether or not x and y are statistically independent.

Problem 2: Gaussian distribution (15 points)

The continuous random variable x has the probability density

$$P(x) = C \exp\left[-\frac{(x-b)^2}{2a^2}\right]$$

for all real x (where a, b, and C are constants).

a) Find the value of the constant C (in terms of a and b) such that the probability density is properly normalized.

b) Compute the average $\langle x \rangle$, the median x_M and the most probable value x_P.

c) Compute the second moment $\langle x^2 \rangle$ and the variance σ_x^2.

Problem 3: Probability of a density fluctuation (12 points)

Consider two identical boxes, A and B.

a) 10 particles are distributed over the two identical boxes A and B at random. Calculate the probabilities $P(4)$ and $P(5)$ for finding exactly $N_A = 4$ and $N_A = 5$ particles in box A, respectively. Calculate the ratio $P(4)/P(5)$.

b) Repeat the calculations for 100 particles. Compare the probabilities for $N_A = 40$ and $N_A = 50$.

c) Repeat the calculations for 1000 particles. Compare the probabilities for $N_A = 400$ and $N_A = 500$.

(For large n the factorial can be approximated by Stirling's formula $n! = \sqrt{2\pi n}\ n^n e^{-n}$)