Physics 4311: Thermal Physics – Final Exam

Friday, May 15, 2026

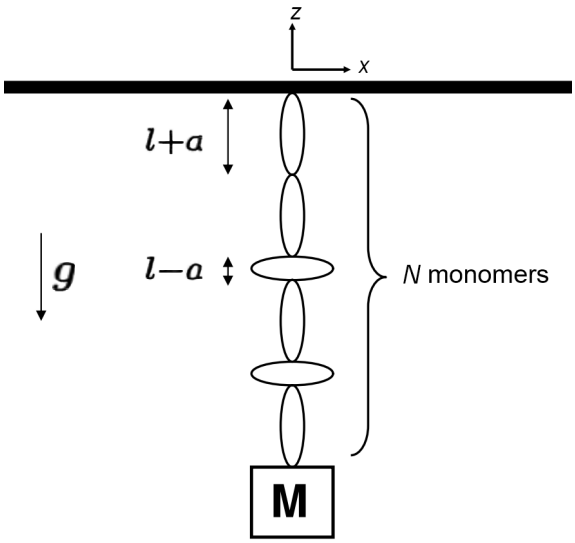
Problem 1: Short questions (10 points each = 30 points)

a) A system with five states is in equilibrium at temperature T. The energies fulfill $E_1 = E_2 > E_3 = E_4 = E_5$. What values does the system's entropy take for $T \to 0$ and $T \to \infty$?

b) A box of volume V contains an ideal gas in equilibrium at temperature T. The gas is now isothermally expanded to volume $2V$. Find the ratio $\langle v_f \rangle / \langle v_i \rangle$ between the average speeds of the particles before and after the expansion.

c) You have two classical ideal gases, one consisting of N atoms, the other consisting of N diatomic molecules. Which gas has the larger heat capacity? Give a short explanation for your answer.

Problem 2: Polymer chain (60 points)



A model of a polymer is formed by connecting N ellipsoid-shaped molecules into a one-dimensional chain. Each molecule has two ways of connecting to the polymer (see figure). It can align either its long axis (length $\ell + a$) or its short axis (length $\ell - a$) with the direction of the chain. One end of the chain is fixed at the ceiling, while the other holds a mass M (see figure). Gravity is acting along the negative z direction.

Neglecting the mass of the monomers, the energy of a given state of the polymer can be written as

$$E = -MgL = -Mg \sum_{i=1}^{N} L_i$$

where L_i is either $\ell + a$ or $\ell - a$, depending on the orientation of monomer i. The system is in equilibrium at temperature T.

a) Calculate the single-monomer partition function Z_1.

b) Find the partition function of the polymer chain, its free energy F, and its internal energy U.

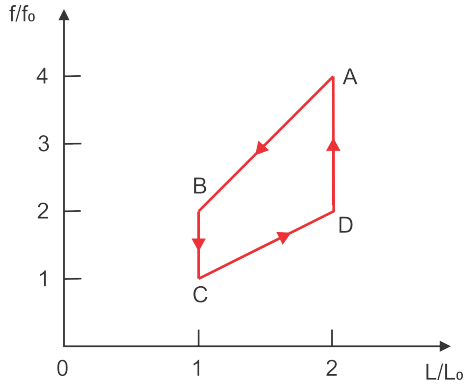
c) Compute the average vertical distance $\langle L \rangle$ between the fixed end of the polymer and the mass M as a function of M, g, ℓ, a, and T.

d) Analyze your result for $\langle L \rangle$ in the limits of $T \to 0$ and $T \to \infty$.

continued on next page

Problem 3: Rubber band heat engine (70 points)

A rubber band has the equation of state $f = \alpha LT$ where L is its length, f is the tension force, T is temperature, and α is a constant. The internal energy is given by $U = C_L T$ where the heat capacity C_L at fixed length is a constant.
Consider a heat engine using this rubber band. The cycle consists of an isothermal shrinking (A $\to$ B), a cooling at constant length (B $\to$ C), an isothermal stretching (C $\to$ D), and a heating a constant length (D $\to$ A).



a) From the figure, find the temperatures at points A, B, C, and D and express them in terms of α, L_0, and f_0.

b) Find the work W_{AB} and heat Q_{AB} during the process A $\to$ B in terms of α, L_0, f_0, and C_L.

c) Find the work W_{BC} and heat Q_{BC} during the process B $\to$ C in terms of α, L_0, f_0, and C_L.

d) Find the work W_{CD} and heat Q_{CD} during the process C $\to$ D in terms of α, L_0, f_0, and C_L.

e) Find the work W_{DA} and heat Q_{DA} during the process D $\to$ A in terms of α, L_0, f_0, and C_L.

f) Find the total work W done on the engine during one cycle.

g) During which of the four processes does the system absorb heat and during which processes does it release heat?

h) The efficiency η of the cycle is defined as work done by the system in one cycle divided by the absorbed heat in one cycle. Compute η and express it in terms of α, L_0, and C_L.

[Hint: For parts b), c), d), and e) carefully think about what quantities change during the process, and which stay constant.]

Problem 4: Nonharmonic classical ideal gas (40 points)

Consider a gas of N non-interacting, indistinguishable, classical particles in a cubic box of linear size L at tenmperature T. The particles have non-harmonic energy-momentum relation; the Hamiltonian reads

$$H = \sum_{i=1}^{N} A|\vec{p}_i|^s \qquad (A, s > 0)$$

a) Calculate the partition function and the free energy [Hint: Work in spherical coordinates.]

b) Calculate the internal energy U and the specific heat C_V at constant volume as functions of T.

c) Calculate the pressure p and find the equation of state (relation between p, V, T).

Hint: $\displaystyle\int_0^\infty dx\, x^2 e^{-ax^s} = \frac{1}{s} a^{-3/s}\, \Gamma(3/s)$ (Γ denotes the Gamma function.)