Physics 4311: Thermal Physics - Final Exam

Monday, May 8, 2023 200 point total

Problem 1: Short questions (10 points each = 30 points)

These questions should not require calculations except, perhaps, one or two lines of math.

a) A system with just two states, with energies E_1 and E_2 which are not identical, is in equilibrium at temperature T. What values does the system's entropy take for $T \to 0$ and $T \to \infty$?

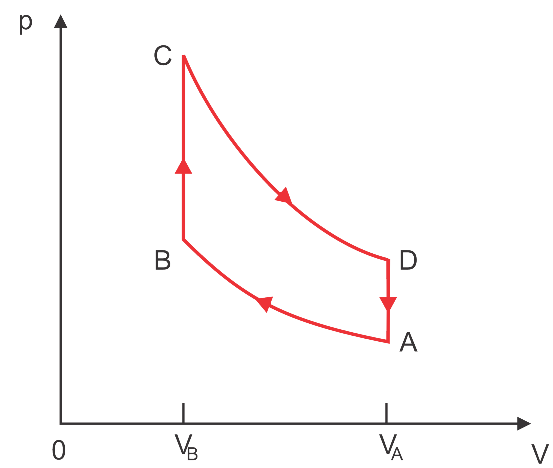
b) Consider a monoatomic classical ideal gas and a diatomic classical ideal gas, both initially at the same pressure, volume, and temperature. Both gases are compressed adiabatically from volume V to volume $V/2$. Which of the gases has the higher temperature after the compression? (Give a brief explanation.)

c) Consider a paramagnetic material whose magnetic susceptibility is governed by the Curie law. Does the magnetization of the material increase or decrease when the material is cooled from a temperature of $300\,\mathrm{K}$ to $100\,\mathrm{K}$ at constant nonzero magnetic field. By what factor does the magnetization change?

Problem 2: Otto cycle (60 points)

The Otto cycle shown in the picture is an idealized version of the process taking place in a gasoline internal combustion engine.

Consider an Otto cycle using a (monoatomic) classical ideal gas of N atoms as working medium. The cycle consists of an adiabatic compression (A $\to$ B), an isochoric heating (B $\to$ C), an adiabatic expansion (C $\to$ D), and an isochoric cooling (D $\to$ A).



a) Use the adiabatic temperature-volume relation to express the temperature T_B in terms of T_A, V_A, and V_B. Also express the temperature T_C in terms of T_D, V_A, and V_B.

b) Find the heat Q_{BC} absorbed in process B $\to$ C and the heat Q_{DA} released in the process D $\to$ A in terms of the temperatures T_A, T_B, T_C, and T_D.

c) Find the work W done by the engine during one cycle. (Use the 1st law!)

d) The efficiency of the cycle is defined as $\eta = |W|/Q_{BC}$. Compute the efficiency in terms of the temperatures T_A, T_B, T_C, and T_D.

e) Show that the efficiency depends on the compression ratio $r = V_A/V_B$ only. Express your result for the efficiency in terms of r.

continued on next page

Problem 3: Ideal gas in spherical trap (60 pts)

Consider a (monoatomic) classical ideal gas of N identical particles of mass m. The particles are held in a spherical trapping potential $V(\vec{r}) = V_0|\vec{r}|/r_0$ (where V_0 and r_0 are positive constants).

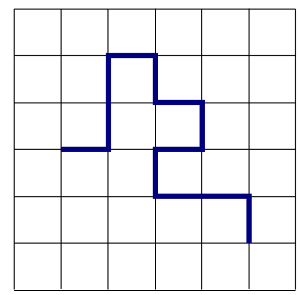
a) Calculate the partition function of the gas at temperature T.

b) Determine the total energy U and compare with the equipartition theorem.

c) Find the particle density $n(\vec{r})$ at position $\vec{r}$ from the origin as a function of T. (Hint: the particle density $n(\vec{r})$ is related to a reduced density of the phase space probability density $\rho(\vec{r}, \vec{p})$.)

d) Find the size of the particle cloud defined as the mean-square distance $\langle|\vec{r}|^2\rangle$ of the particles from the origin as a function of T.

Problem 4: Polymer on a lattice (50 points)



A polymer can be modeled as a path of $N + 1$ identical segments on a square lattice (see picture) connected by N independent joints (lattice sites). At each of the N joints, the polymer can either go straight, or it can bend by 90 degrees left or right. (Different segments of the polymer do not interact with each other, i.e., the path can intersect itself.) A straight joint has zero energy while a right-angle joint has positive energy ϵ. Assume that one end of the polymer is at a fixed position.

a) Find the partition function of this polymer as function of temperature T, the joint energy ϵ, and the number of joints N.

b) Calculate the internal energy U of the polymer.

c) Find the average number $\langle N_{st}\rangle$ of straight joints as a function of temperature T, ϵ, and N.

d) How does $\langle N_{st}\rangle$ behave for $T \to 0$ and $T \to \infty$?

$\int_{-\infty}^{\infty} e^{-\frac{1}{2}(x-x_0)^2/\sigma^2} = (2\pi\sigma^2)^{1/2}$

$\int_0^{\infty} dx\, xe^{-ax} = 1/a^2, \qquad \int_0^{\infty} dx\, x^2 e^{-ax} = 2/a^3, \qquad \int_0^{\infty} dx\, x^3 e^{-ax} = 6/a^4,$

$\int_0^{\infty} dx\, x^4 e^{-ax} = 24/a^5$