1) $dU = T\,dS + f\,dL \qquad \left(\frac{\partial T}{\partial L}\right)_S = \left(\frac{\partial f}{\partial S}\right)_L$

$dF = -S\,dT + f\,dL \qquad \left(\frac{\partial S}{\partial L}\right)_T = -\left(\frac{\partial f}{\partial T}\right)_L$

$dH = T\,dS - L\,df \qquad \left(\frac{\partial T}{\partial f}\right)_S = -\left(\frac{\partial L}{\partial S}\right)_f$

$dG = -S\,dT - L\,df \qquad \left(\frac{\partial S}{\partial f}\right)_T = \left(\frac{\partial L}{\partial T}\right)_f$

2) heat balance $\quad Q_{loss} = |Q_h|$

$$A(T_h - T_e) = \frac{E}{\eta} = \frac{E\,T_h}{T_h - T_e}$$

$$(T_h - T_e)^2 = \frac{E}{A} T_h$$

$$T_h^2 - 2T_hT_e - \frac{E}{A}T_h + T_e^2 = 0$$

$$T_h = T_e + \frac{E}{2A} \pm \sqrt{\left(T_e + \frac{E}{2A}\right)^2 - T_e^2}$$

choose + so that $T_h > T_e$

$$T_h = T_e + \frac{E}{2A} + \sqrt{\left(\frac{E}{2A}\right)^2 + \frac{E}{A}T_e}$$

3) $F = -S\,dT - m\,dB$

$$\left(\frac{\partial S}{\partial B}\right)_T = \left(\frac{\partial m}{\partial T}\right)_B$$

$$\left(\frac{\partial S}{\partial B}\right)_T = \left(\frac{\partial}{\partial T}\frac{CB}{T}\right)_B = -\frac{CB}{T^2}$$

4) a) $Q_{AB} = C_p(T_B - T_A)$ $\left(\text{from } \left(\frac{\delta Q}{\partial T}\right)_p = C_p\right)$

b) $Q_{CD} = C_p(T_D - T_C)$

c) 1st law $W + Q_{AB} + Q_{CD} = 0$

$$W = -C_p(T_B - T_A) - C_p(T_D - T_C)$$

d) $$\eta = \frac{-W}{Q_{AB}} = 1 - \frac{T_C - T_D}{T_B - T_A}$$

On adiabat $D \to A$: $p_1 V_D^{\gamma} = p_2 V_A^{\gamma}$, $V = \frac{Nk_BT}{p}$

$$p_1^{1-\gamma} T_D^{\gamma} = p_2^{1-\gamma} T_A^{\gamma}$$

analogously $p_1^{1-\gamma} T_C^{\gamma} = p_2^{1-\gamma} T_B^{\gamma}$

$$\eta = 1 - \frac{T_B\left(\frac{p_2}{p_1}\right)^{\frac{1-\gamma}{\gamma}} - T_A\left(\frac{p_2}{p_1}\right)^{\frac{1-\gamma}{\gamma}}}{T_B - T_A} = 1 - \left(\frac{p_2}{p_1}\right)^{\frac{1-\gamma}{\gamma}}$$