Physics 4311: Thermal Physics – Test 2 Solution

Friday, April 10, 2026

Problem 1: TD potentials and Maxwell relations (42 points)

a) $H = U - fL \quad , \qquad dH = TdS - Ldf$
$F = U - TS \quad , \qquad dF = -SdT + fdL$
$G = U - TS - fL \quad , \qquad dG = -SdT - Ldf$

b) from U: $\left(\frac{\partial f}{\partial S}\right)_L = \left(\frac{\partial T}{\partial L}\right)_S$
from H: $\left(\frac{\partial L}{\partial S}\right)_f = -\left(\frac{\partial T}{\partial f}\right)_S$
from F: $\left(\frac{\partial S}{\partial L}\right)_T = -\left(\frac{\partial f}{\partial T}\right)_L$
from G: $\left(\frac{\partial S}{\partial f}\right)_T = \left(\frac{\partial L}{\partial T}\right)_f$

Problem 2: Stretching a rubber band (60 points)

a) $\delta W = fdL$
$\Delta W = \int_{L_0}^{2L_0} fdL = \int_{L_0}^{2L_0} \alpha LTdL = \frac{3}{2}\alpha T L_0^2$

b) $dU = \delta Q + \delta W, \quad$ isothermal $dU = 0, \quad \Delta Q = -\Delta W = -\frac{3}{2}\alpha T L_0^2$

c) $dS = \delta Q/T, \quad \Delta S = \Delta Q/T = -\frac{3}{2}\alpha L_0^2$

Problem 3: Ideal refrigerator (48 points)

The heat egine, running backwards, must remove $Q_{loss} + Q_B$ from the low tempearture reservoir: $Q_l = Q_{loss} + Q_B$

Use $(Q_h + Q_l)/Q_h = 1 - T_l/T_h \quad \Rightarrow \quad Q_l/Q_h = -T_l/T_h \quad \Rightarrow \quad Q_h = -(T_h/T_l)Q_l$

$$W = -(Q_h + Q_l) = \left(\frac{T_h}{T_l} - 1\right) Q_l = \left(\frac{T_h}{T_l} - 1\right)(Q_{loss} + Q_B)$$

$$W = A\frac{(T_h - T_l)^2}{T_l} + Q_B\frac{T_h - T_l}{T_l}$$