Physics 4311: Thermal Physics – Test 2

Friday, April 10, 2026

Problem 1: TD potentials and Maxwell relations (42 points)

Consider an elastic rod or rubber band of length L under tension force f, for which the first law reads $dU = T\,dS + f\,dL$.

a) Derive the enthalpy, Helmholtz free energy, and Gibbs free energy as well as their total differentials by performing the appropriate Legendre transformations.

b) Derive all four Maxwell relations for this system.

Problem 2: Stretching a rubber band (60 points)

A rubber band has the equation of state $f = \alpha LT$ where L is its length, f is the tension force, T is temperature, and α is a constant. The internal energy is given by $U = C_L T$ where the heat capacity C_L at fixed length is a constant.

a) Compute the work done on the rubber band when it is stretched isothermally from length L_0 to $2L_0$.

b) Find the heat flowing into the rubber band when it is stretched isothermally from L_0 to $2L_0$.

c) Determine the change in entropy when the rubber band is stretched isothermally from L_0 to $2L_0$.

Problem 3: Ideal refrigerator (48 points)

An ideal refrigerator consists of a Carnot cycle (running backwards). Over the period of an hour, it removes heat Q_l from the interior of the device at the lower temperature T_l and discharges heat Q_h into the house at the (higher) room temperature T_h, consuming electric energy (work) W. The amount of heat leaking into the refrigerator through its walls per hour is $Q_{loss} = A(T_h - T_l)$ where A is a constant. In addition, a light bulb is switched on inside the refrigerator, generating heat Q_B per hour. The refrigerator has run for a while and has reached a steady state.

a) Derive an expression for the energy W required to run the refrigerator (in the steady state) as a function of T_l, T_h, Q_B, and A.

Hint: You may start from (or use) the efficiency of a Carnot cycle running forward (as heat engine): $-W/Q_h = (Q_h + Q_l)/Q_h = 1 - T_l/T_h$.