Tuesday, Apr 11, 2023

150 point total + 12 BONUS

Problem 1: Maxwell relations (24 points)

Find all four Maxwell relations for a dielectric material for which the first law reads dU = T dS - P dE. (You can treat the polarization P and the electric field E as scalars.)

Problem 2: Entropy of rubber band (30 points)

An idealized rubber band has the equation of state f = ALT, where L is its length, f is the tension force, T is the temperature, and A is a constant. The internal energy is given by $U = C_L T$ where C_L is a constant. Starting from the first law, find the change in entropy as the rubber band is stretched from length L_0 at temperature T_0 to length L at temperature T. (Express the answer in terms of L_0, L, T_0, T and the constants).

Problem 3: Ideal refrigerator (36 points)

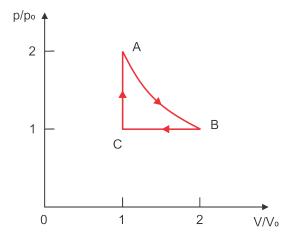
An ideal refrigerator consists of a Carnot cycle (running backwards). It removes heat Q_l from the inside at the lower temperature T_l and discharges heat Q_h to the outside at the (higher) room temperature T_h , consuming electric energy (work) E.

- a) The refrigerator runs continuously, and the inside temperature has reached a steady state. Derive a relation for the inside temperature T_l in terms of T_h , Q_l and E.
- b) A 100 Watt light bulb is left burning inside the refrigerator which consumes electric power of 50 Watt when running. If the outside temperature is 300 K, what is the inside temperature in the steady state? (Neglect heat leaking into the refrigerator from the outside.)

Hint: You may start from the efficiency of a Carnot cycle running forward (as heat engine): $|W|/Q_h = -W/Q_h = 1 - T_l/T_h$.

Problem 4: Heat engine (60 points + 12 BONUS points)

A heat engine uses an ideal gas of N atoms as working medium. It undergoes the cycle shown in the figure which consists of an isothermal expansion $(A \rightarrow B)$, an isobaric compression $(B \rightarrow C)$, and an isochoric heating $(C \rightarrow A)$.



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- a) Find the temperatures T_A , T_B , and T_C at points A, B, and C in terms of p_0 , V_0 , and N.
- b) Compute the change in internal energy ΔU , the work ΔW , and the absorbed heat ΔQ for the isothermal process $A \to B$ (in terms of p_0, V_0).
- c) Compute ΔU , ΔW , and ΔQ for the isobaric process $B \to C$ (in terms of p_0, V_0).
- d) Compute ΔU , ΔW , and ΔQ for the isochoric process $C \to A$ (in terms of p_0, V_0).
- e) (BONUS) The efficiency of the cycle is defined as $\eta = |W|/Q_{in}$, where Q_{in} is the total "input" heat that is absorbed by the engine (i.e. heat terms that are positive). Compute the efficiency.
- f) (BONUS) What is the efficiency of a Carnot engine running between T_A and T_C ? Compare.