Physics 4311: Thermal Physics - Exam 2

Thursday, Apr 11, 2024 150 point total

Problem 1: Maxwell relations of elastic rod (40 points)

Find all four Maxwell relations for an elastic rod for which the first law reads $dU = T\,dS + f\,dL$ where f is the tension force and L is the length of the rod.

Problem 2: Heat pump (30 points)

A house is heated by an ideal heat pump consisting of a Carnot cycle (running backwards). Over the period of an hour, it removes heat Q_l from the outside at the lower temperature T_l and discharges heat Q_h into the house at the (higher) room temperature T_h, consuming electric energy (work) E. The amount of heat leaking out of the house through walls and windows per hour is $Q_{\text{loss}} = A(T_h - T_l)$ where A is a constant.

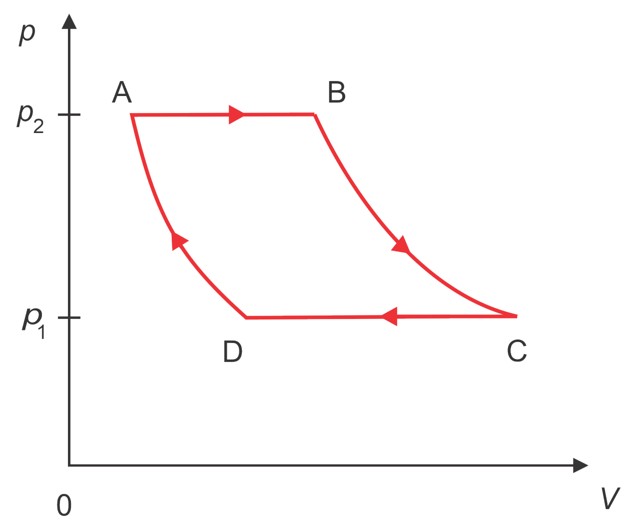
Derive an expression for the temperature T_h inside the house as a function of T_l, E , and A. [Hint: You may start from the efficiency of a Carnot cycle running forward (as heat engine): $|W|/Q_h = -W/Q_h = 1 - T_l/T_h$.]

Problem 3: Entropy in a paramagnet (20 points)

A paramagnetic material at temperature T has the equation of state $m = CB/T$ where m is the magnetization and B is the magnetic field (induction). Derive an an expression for the change in entropy with field at fixed temperature, $(\partial S/\partial B)_T$ for this material. [Hint: Derive and use an appropriate Maxwell relation.]

Problem 4: Isobaric-adiabatic cycle (60 points)

An ideal gas fulfills the equation of state $pV = Nk_BT$. It has constant heat capacity c_p at fixed pressure and an adiabatic index γ. The gas undergoes the cycle shown in the figure which consists of an isobaric expansion at pressure p_2 (A $\to$ B), an adiabatic expansion (B $\to$ C), an isobaric compression at pressure p_1 (C $\to$ D), and an adiabatic compression (D $\to$ A).



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a) Compute the heat Q_{AB} absorbed during the isobaric process A $\to$ B in terms of c_p and the temperatures T_A and T_B at points A and B, respectively.

b) Compute the heat Q_{CD} emitted during the isobaric process C $\to$ D in terms of c_p and the temperatures T_C and T_D at points C and D, respectively.

c) Express the work done on the system during one cycle in terms of the answers to parts a and b.

d) Compute the efficiency of the cycle as a heat engine and express it in terms of the pressures p_1 and p_2 (and γ) only. [Hint: It may be helpful to establish a relation between p and T for each of the adiabatic processes, B $\to$ C and D $\to$ A .]