Physics 4311: Thermal Physics – Test 1

Friday, Feb 27, 2026

Problem 1: Short questions (15 points each = 45 points)

a)

$$P = \binom{4}{2}(1/4)^2\,(3/4)^2 = 27/128$$

b)

$$\Phi = \frac{1}{4}n\langle v\rangle \sim \frac{1}{\sqrt{m}}$$

$$\Phi_{Ne}/\Phi_{Ar} = \sqrt{m_{Ar}/m_{Ne}} = \sqrt{2}$$

c)

$$\lambda = \frac{1}{\sqrt{2}n\sigma}$$

n stays constant upon changing $T \Rightarrow \lambda$ unchanged

Problem 2: Molecules on surface (60 points)

a), b), c)

$$p_{up} = \frac{e^{-\beta\epsilon}}{2+e^{-\beta\epsilon}}\ , \quad p_x = \frac{1}{2+e^{-\beta\epsilon}}\ , \quad p_y = \frac{1}{2+e^{-\beta\epsilon}}$$

d) $T \to 0,\ \beta \to \infty\ \Rightarrow\ p_{up} \to 0$

e) $T \to \infty,\ \beta \to 0\ \Rightarrow\ p_{up} \to 1/3$

a)

$$\langle E\rangle = \frac{\epsilon e^{-\beta\epsilon}}{2+e^{-\beta\epsilon}}$$

b) $T \to 0,\ \beta \to \infty\ \Rightarrow\ \langle E\rangle \to 0$

c) $T \to \infty,\ \beta \to 0\ \Rightarrow\ \langle E\rangle \to \epsilon/3$

continued on next page

Problem 3: Two-dimensional ideal gas (45 points)

a)

$$g(v_x, v_y) = \frac{m}{2\pi k_B T} e^{-\beta m \vec{v}^2/2}$$

b)

$$\langle E \rangle = \frac{m}{2}(\langle v_x^2 \rangle + \langle v_y^2 \rangle) = k_B T$$

c)

$$\begin{aligned} f(E) &= \int dv_x dv_y \, g(v_x, v_y) \delta(E - m(v_x^2 + v_y^2)/2) \\ &= \frac{m}{2\pi k_B T} \int v dv d\phi e^{-\frac{1}{2}mv^2/k_B T} \delta(E - mv^2/2) \\ &= \frac{m}{k_B T} \int_0^\infty \frac{dx}{m} e^{-\beta x} \delta(E - x) \\ &= \frac{1}{k_B T} \, e^{-E/k_B T} \end{aligned}$$

$$\int_{-\infty}^{\infty} dx \; e^{-\frac{1}{2}x^2/\sigma^2} = (2\pi\sigma^2)^{1/2} \qquad \int_{-\infty}^{\infty} dx \; x^2 e^{-\frac{1}{2}x^2/\sigma^2} = (2\pi\sigma^2)^{1/2}\sigma^2$$

$$\int_0^\infty dx \, x e^{-ax} = 1/a^2, \qquad \int_0^\infty dx \, x^2 e^{-ax} = 2/a^3, \qquad \int_0^\infty dx \, x^3 e^{-ax} = 6/a^4, \qquad \int_0^\infty dx \, x^4 e^{-ax} = 24/a^5$$