Physics 4311: Thermal Physics - Exam 1

Tuesday, Feb 28, 2023 150 point total

Problem 1: Short questions (10 points each = 40 points)

a) A particle performs a random walk consisting of 5 independent steps. In each step it moves a distance a either to the left or to the right with equal probability. Find the probability that the particle ends up a distance $3a$ to the right of its starting point at the end of the walk.

b) A box contains an ideal gas consisting of Neon atoms (atomic mass 20) and Argon atoms (atomic mass 40). The gas is in equilibrium at temperature T. Find the ratio $\langle v\rangle_{Ne}/\langle v\rangle_{Ar}$ between the average speeds of the Neon and Argon atoms.

c) Consider an ideal gas in equilibrium. Using symmetry arguments, find which of the following averages involving velocity components v_x, v_y, v_z are zero: $\langle v_x\rangle$, $\langle v_y^2\rangle$, $\langle v_x v_z\rangle$, $\langle v_y^2 v_z\rangle$, $\langle v_x^2 v_y^4\rangle$

d) An ideal gas is compressed from volume $2V$ to volume V. Which process leads to a larger increase in pressure, an isothermal compression or an adiabatic compression. Give a short explanation for your answer.

Problem 2: High-speed molecules (20 points)

Consider a two-dimensional ideal gas of molecules of mass m at temperature T. The two-dimensional Maxwell velocity distribution reads

$$P(v_x, v_y) = \frac{m}{2\pi k_B T} e^{-\frac{m}{2}(v_x^2+v_y^2)/k_B T}.$$

Find the probability of a molecule having a speed larger than twice the root-mean square velocity of $\sqrt{2k_BT/m}$. Hint: Go to polar coordinates!

Problem 3: Absorbed atom (40 points)

An atom can be absorbed on the surface of a solid in two different lattice positions. In the first position, it has an energy $E_1 = -\epsilon$ with $\epsilon > 0$. In the second position, its energy is $E_2 = 0$. The system is in thermal equilibrium at temperature T.

a) Compute the probability for the absorbed atom to be in position 1.

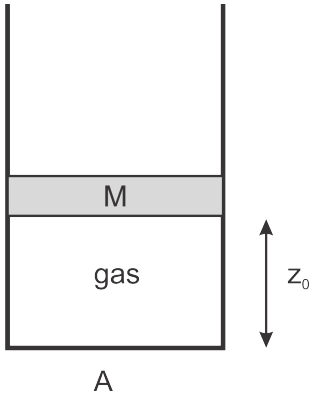
b) Compute the probability for the absorbed atom to be in position 2.

c) What is the average energy $\langle E\rangle$ of the absorbed atom as function of the temperature?

d) Determine the limiting values of $\langle E\rangle$ for $T \to 0$ and $T \to \infty$.

continued on next page

Problem 4: Movable piston (50 points)



An ideal gas of N atoms is contained in a cylindrical vessel of cross section A with a piston of mass M on top. The piston can move up and down, keeping the pressure constant, but the gas cannot escape. The device is surrounded by vacuum. Initially, the cylinder is at rest, the gas is in equilibrium at temperature T_0, and the vertical position of the cylinder is z_0.

a) Consider the forces acting on the piston, and find the initial pressure inside the gas.

b) The gas is now heated up slowly, lifting the piston. Find the temperature T of the gas, when the height of the cylinder reaches $z = 2z_0$.

c) Compute the work done by the gas during the expansion. (Does the pressure change in this process?)

d) Find the change in the internal energy of the gas in this process.

e) How much heat has to be provided to the gas in this process? Express the answer in terms of N and T_0.