Physics 4311: Thermal Physics - Exam 1

Tuesday, Feb 29, 2024 150 point total

Problem 1: Short questions (10 points each = 40 points)

a) You have 5 independent magnetic atoms. The magnetic moment of each atom points "up" with probability 2/3 or "down" with probability 1/3. Find the probability that the moments of 3 of the 5 atoms point "down" while the moments of the other 2 atoms point "up".

b) Consider two boxes, one containing an ideal gas of He atoms (atomic mass 4). The other box contains an ideal gas of Ar atoms (atomic mass 40). The root-mean-square velocities of the atoms in the two gases are identical. Find the ratio of temperatures of the two gases, $T_{\rm Ar}/T_{\rm He}$.

c) A box contains an ideal gas of atoms of radius a, number density $N/V = n$, and temperature T. the temperature is now raised from T to $3T$. Does the mean free path of the atoms (i) increase, (ii) decrease, or (iii) stay the same? (circle one!)

d) Starting from the equation of state, derive the isothermal compressibility of the ideal gas, $\kappa_T = -(1/V)(\partial V/\partial p)_T$, and express it in terms of p.

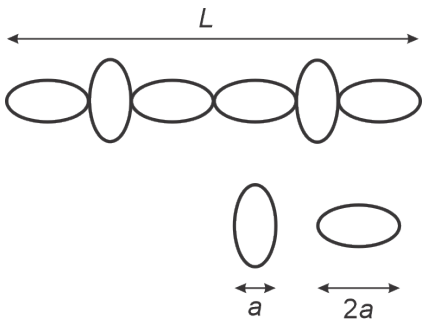
Problem 2: 2D Maxwell distribution (20 points)

Consider a two-dimensional ideal gas of molecules of mass m at temperature T. The two-dimensional Maxwell velocity distribution reads

$$P(v_x, v_y) = \frac{m}{2\pi k_B T} e^{-\frac{m}{2}(v_x^2+v_y^2)/k_B T}.$$

Starting from this Maxwell distribution, derive the (properly normalized) probability density $f(v)$ for the speed $v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$. (Hint: Go to polar coordinates.)

Problem 3: One-dimensional polymer (40 points)



A one-dimensional polymer is formed by connecting N ellipsoid-shaped molecules into a one-dimensional chain. Each molecule has two ways of connecting to the polymer (as shown in the figure). It can align either its long axis (length $2a$) or its short axis (length a) with the direction of the polymer chain. A molecule connected along the long axis has energy $E_1 = 0$, a molecule connected along the short axis has energy $E_2 = \epsilon$ with $\epsilon > 0$. The polymer is in equilibrium at temperature T

a) Calculate the probability p_l for a molecule to be connected along the long axis.

b) Calculate the probability p_s for a molecule to be connected along the short axis.

c) Calculate the average length L of the polymer as a function of T and N.

d) What is value of the average length in the limit $T \to 0$?

e) What is value of the average length in the limit $T \to \infty$?

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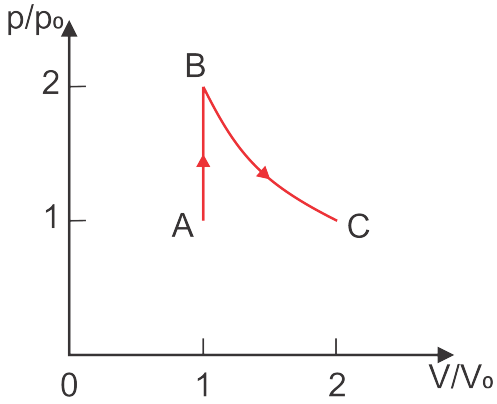
Problem 4: Gas expansion (50 points)

An ideal gas of N atoms is taken quasi-statically from point A to B (at constant volume) and then from B to C (at constant temperature) as shown in the pressure-volume diagram. Express all answers in terms of N, k_B, p_0, and V_0.



a) Find the temperatures T_A, T_B and T_C at points A, B, and C.

b) How much work is done on the gas from A to B?

c) How much heat is flowing into the gas from A to B?

d) How much work is done on the gas during the isothermal expansion from B to C?

e) How much heat is flowing into the gas from B to C?